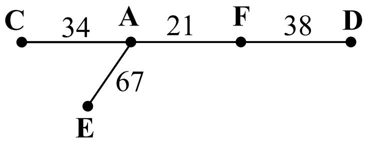
# June 2009
# 6690 Decision Mathematics D2
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| Q1 (a) | There are more tasks than people. | B1 (1) |
| (b) | Adds a row of zeros | B1 (1) |
| (c) | $\begin{bmatrix} 15 & 11 & 14 & 12 \\ 13 & 8 & 17 & 13 \\ 14 & 9 & 13 & 15 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 0 & 3 & 1 \\ 5 & 0 & 9 & 5 \\ 5 & 0 & 4 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \rightarrow \begin{bmatrix} 3 & 0 & 2 & 0 \\ 4 & 0 & 8 & 4 \\ 4 & 0 & 3 & 5 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | B1;M1A1 |
| | Either $\begin{bmatrix} 3 & 3 & 2 & 0 \\ 1 & 0 & 5 & 1 \\ 1 & 0 & 0 & 2 \\ 0 & 4 & 0 & 0 \end{bmatrix}$ | M1 A1 |
| | Or $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 6 & 4 \\ 2 & 0 & 1 & 5 \\ 0 & 3 & 0 & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 5 & 3 \\ 1 & 0 & 0 & 4 \\ 0 & 4 & 0 & 2 \end{bmatrix}$ | |
| | J – 4, M – 2, R – 3, (D – 1) | A1 (6) |
| (d) | Minimum cost is (£)33. | B1 (1) |
| | | **[9]** |

| Question Number | Scheme | Marks |
|---|---|---|
| Q2 | | |
| (a) | In the classical problem each vertex must be visited **only** once.<br>In the practical problem each vertex must be visited **at least** once. | B2, 1, 0<br>(2) |
| (b) | A F D B E C A {1 4 6 3 5 2 }<br>21 + 38 + 58 + 36 + 70 + 34 = 257 | M1 A1<br>A1 (3) |
| (c) | 257 is the better upper bound, it is lower. | B1ft (1) |
| (d) | R.M.S.T.  | M1 A1 |
| | Lower bound is 160 + 36 + 58 = 254 | M1A1 (4) |
| (e) | Better lower bound is 254, it is higher | B1ft |
| (f) | $254 < \text{optimal} \leq 257$ | B1 (2) |
| | | **[12]** |
| | **Notes:**<br>(a) 1B1: Generous, on the right lines bod gets B1<br>2B1: cao, clear answer.<br>(b) 1M1:Nearest Neighbour each vertex visited once (condone lack of return to start)<br>1A1: Correct route cao – must return to start.<br>2A1: 257 cao<br>(c) 1B1ft: ft their lowest.<br>(d) 1M1: Finding correct RMST (maybe implicit) 160 sufficient<br>1A1: cao tree or 160.<br>2M1:Adding 2 least arcs to B, 36 and 58 only<br>2A1: 254<br>(e) 1B1ft: ft their highest<br>(f) 1B1: cao | |

| Question Number | Scheme | Marks |
|---|---|---|

**Q3**

(a)

Row minima {-5, -4, -2} row maximin = -2

Column maxima {1, 6, 13} col minimax = 1 — M1 A1

$-2 \neq 1$ therefore not stable. — A1 (3)

(b)

Column 1 dominates column 3, so column 3 can be deleted. — B1 (1)

(c)

| | A plays 1 | A plays 2 | A plays 3 |
|---|---|---|---|
| B plays 1 | 5 | -1 | 2 |
| B plays 2 | -6 | 4 | -3 |

B1 B1 (2)

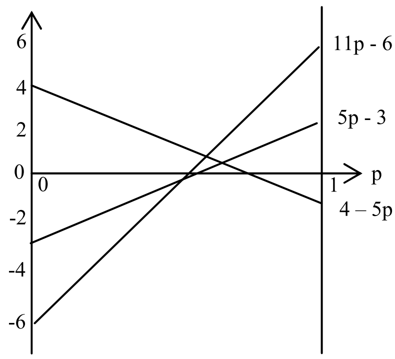
(d)

Let B play row 1 with probability p and row 2 with probability (1-p)

If A plays 1, B's expected winnings are 11p – 6 — M1 A1

If A plays 2, B's expected winnings are 4 – 5p

If A plays 3, B's expected winnings are 5p – 3




M1 A1

$$5p - 3 = 4 - 5p$$ M1

$$10p = 7$$

$$p = \frac{7}{10}$$

B should play 1 with a probability of 0.7

2 with a probability of 0.3 — A1

and never play 3

The value of the game is 0.5 to B — A1 (7)

**[13]**

| Question Number | Scheme | Marks |
|---|---|---|
| Q4 (a) | Value of cut $C_1 = 34$; Value of cut $C_2 = 45$ | B1; B1 (2) |
| (b) | S B F G T or S B F E T – value 2<br>Maximum flow = 28 | M1 A1<br>A1=B1 (3)<br>**[5]** |
| | **Notes:**<br>(a) 1B1: cao<br>2B1: cao<br>(b) 1M1: feasible flow-augmenting route and a value stated<br>1A1: a correct flow-augmenting route and value<br>1A1= B1: cao | |
| Q5 (a) | $x = 0, \; y = 0, \; z = 2$ | B2,1,0 (2) |
| (b) | $P - 2x - 4y + \frac{5}{4}r = 10$ | M1 A1 (2)<br>**[4]** |
| | **Notes:**<br>(a) 1B1: Any 2 out of 3 values correct<br>2B1: All 3 values correct.<br>(b) 1M1: One equal sign, modulus of coefficients correct. All the right ingredients.<br>1A1: cao – condone terms of zero coefficient | |

| Question Number | Scheme | Marks |
|---|---|---|

**Q6**

**(a)** The supply is equal to the demand — B1 (1)

**(b)** — B1 (1)

| | A | B | C |
|---|---|---|---|
| X | 16 | 6 | |
| Y | | 9 | 8 |
| Z | | | 15 |

**(c)**

| | A | B | C |
|---|---|---|---|
| X | 16- θ | 6+ θ | |
| Y | | 9- θ | 8+ θ |
| Z | θ | | 15- θ |

M1 A1

Value of θ = 9, exiting cell is YB — A1 (3)

**(d)**

| | | A | B | C |
|---|---|---|---|---|
| | | 17 | 8 | 20 |
| 0 | X | 7 | 15 | |
| -5 | Y | | | 17 |
| -11 | Z | 9 | | 6 |

M1 A1

XC = 7 – 0 – 20 = -13
YA = 16 + 5 – 17 = 4
YB = 12 + 5 – 8 = 9
ZB = 10 + 11 – 8 = 13

A1 (3)

| | A | B | C |
|---|---|---|---|
| X | 7- θ | 15 | θ |
| Y | | | 17 |
| Z | 9+ θ | | 6- θ |

Value of θ = 6, entering cell XC, exiting cell ZC — M1 A1

| | A | B | C |
|---|---|---|---|
| X | 1 | 15 | 6 |
| Y | | | 17 |
| Z | 15 | | |

A1 (3)

Cost (£) 524 — B1 (1)

**[12]**

| Question Number | Scheme | Marks |
|---|---|---|
| Q7 (a) | | |

| Stage | State (in £1000s) | Action (in £1000s) | Dest. (in £1000s) | Value (in £1000s) | Marks |
|---|---|---|---|---|---|
| | 250 | 250 | 0 | 300* | |
| 1 | 200 | 200 | 0 | 240* | |
| | 150 | 150 | 0 | 180* | |
| | 100 | 100 | 0 | 120* | |
| | 50 | 50 | 0 | 60* | |
| | 0 | 0 | 0 | 0* | |
| | 250 | 280 | 0 | 200 + 0 = 280 | |
| | | 200 | 50 | 235 + 60 = 295 | |
| | | 150 | 100 | 190 + 120 = 310* | |
| | | 100 | 150 | 125 + 180 = 305 | 1M1 A1 |
| | | 50 | 200 | 65 + 240 = 305 | |
| | | 0 | 250 | 0 + 300 = 300 | |
| 2 | 200 | 200 | 0 | 235 + 0 = 235 | |
| | | 150 | 50 | 190 + 60 = 250* | |
| | | 100 | 100 | 125 + 120 = 245 | A1 |
| | | 50 | 150 | 65 + 180 = 245 | |
| | | 0 | 200 | 0 + 240 = 240 | |
| | 150 | 150 | 0 | 190 + 0 = 190* | 2M1 |
| | | 100 | 50 | 125 + 60 = 185 | |
| | | 50 | 100 | 65 + 120 = 185 | A1 |
| | | 0 | 150 | 0 + 180 = 180 | |
| | 100 | 100 | 0 | 125 + 0 = 125* | A1 |
| | | 50 | 50 | 65 + 60 = 125* | |
| | | 0 | 100 | 0 + 120 = 120 | |
| | 50 | 50 | 0 | 65 + 0 = 65* | |
| | | 0 | 50 | 0 + 60 = 60 | |
| | 0 | 0 | 0 | 0 + 0 = 0* | 3M1 A1ft |
| 3 | 250 | 250 | 0 | 300 + 0 = 300 | |
| | | 200 | 50 | 230 + 65 = 295 | |
| | | 150 | 100 | 170 + 125 = 295 | |
| | | 100 | 150 | 110 + 190 = 300 | |
| | | 50 | 200 | 55 + 250 = 305 | |
| | | 0 | 250 | 0 + 310 = 310* | |

Maximum income £310 000 — B1

| Scheme | 1 | 2 | 3 |
|---|---|---|---|
| Invest (in £1000s) | 100 | 150 | 0 |

B1 (10)

(b)

Stage: Scheme being considered — B1
State: Money available to invest — B1
Action: Amount chosen to invest — B1 (3)

**[13]**

| Question Number | Scheme | Marks |
|---|---|---|
| Q8 | E.g. Add 6 to make all elements positive $\begin{bmatrix} 4 & 14 & 5 \\ 13 & 10 & 3 \\ 7 & 1 & 10 \end{bmatrix}$ | B1 |
| | Let Laura play 1, 2 and 3 with probabilities $p_1$, $p_2$ and $p_3$ respectively<br>Let V = value of game + 6 | B1 |
| | e.g.<br>Maximise P = V<br>Subject to: | B1 |
| | $V - 4p_1 - 13p_2 - 7p_3 \le 0$<br>$V - 14p_1 - 10p_2 - p_3 \le 0$<br>$V - 5p_1 - 3p_2 - 10p_3 \le 0$<br>$p_1 + p_2 + p_3 \le 1$<br>$p_1, p_2, p_3 \ge 0$ | M1<br>A3,2ft,1ft ,0<br><br>(7) |
| | **Notes:**<br>1B1: Making all elements positive<br>2B1: Defining variables<br>3B1: Objective, cao word and function<br>1M1: At least one constraint in terms of their variables, must be going down columns. Accept = here.<br>1A1ft: ft their table. One constraint in V correct.<br>2A1ft: ft their table. Two constraints in V correct.<br>3A1: CAO all correct . | **[7]** |
| | **Alt using $x_i$ method**<br><br>Now additionally need: let $x_i = \frac{p_i}{v}$ for 2B1<br><br>minimise $(P) = x_1 + x_2 + x_3 = \frac{1}{v}$<br><br>subject to:<br>$4x_1 + 13x_2 + 7x_3 \ge 1$<br>$14x_1 + 10x_2 + x_3 \ge 1$<br>$5x_1 + 3x_2 + 10x_3 \ge 1$<br>$x_i \ge 0$ | |