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“William Yolland’s notable contribution to the
Ordnance Survey 1841-1852”

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The Charles Close Society was founded in 1980 to bring together all those with an interest in the maps and history of the Ordnance Survey of Great Britain and its counterparts in the island of Ireland. The Society takes its name from Colonel Sir Charles Arden-Close, OS Director General from 1911 to 1922, and initiator of many of the maps now sought after by collectors.

The Society publishes a wide range of books and booklets on historic OS map series and its journal, *Sheetlines*, is recognised internationally for its specialist articles on Ordnance Survey-related topics.

William Yolland's notable contribution to the Ordnance Survey 1841-1852

David L Walker

Michael Cory's thorough and readable article on the Lough Foyle baseline (*Sheetlines* 130) was nicely complemented by Karen Rann's delightful insight into early contouring on the other side of the Inishowen Peninsula. But Michael Cory unsurprisingly skips William Yolland's lengthy account within his Lough Foyle paper of 'the various methods of computation followed on the Ordnance Survey.'¹

Yolland's account is crucial in providing a full description of his basis for calculations after 1845 of the latitude and longitude of the principle [*sic*] and secondary points of the triangulation; of some Scottish initial points for the county meridians; and of the corners of Ordnance Survey one-inch maps and six-inch plans. Importantly, it also includes his surprisingly forthright account of mistakes made in earlier years.

The initial triangulation of England, Wales and Southern Scotland was related to nine different meridians, which resulted in the fragmented network shown in Figure 1.² This was unified as a result of the innovations managed by Yolland in the 1840s. These created a single network, related to Airy's figure of the earth, that remained in use for another century. This provided the basis for AR Clarke's adjusted network, but this was not adopted for the survey.

This article summarises and illustrates the significant and enduring changes managed by Yolland over this period. It is documented from the substantial but incomplete records that survive in The National Archives, and takes account of other research by members of the Charles Close Society.³

Doubts within the Ordnance in the 1830s

Thomas Colby, intent on overtaking the French, seemed ready to ignore the figure of the earth based on the French and Indian baselines published by William Lambton in 1818; the defect in Ordnance latitudes found by John Tiarks in 1824; doubts over any reliance on observations of Polaris expressed by Captain Kater in 1828; and further work on the figure of the earth published by Professor George Airy in 1830.

However, Lt Hastings Murphy of the Ordnance apparently decided in 1830 (with the knowledge of Captain Robe) to recalculate latitudes and longitudes provided to the Admiralty to accord with Lambton's figure of the earth. On this basis he provided three positions near the Scottish border that differed by about 30 seconds in longitude from those published for the Ordnance in 1811. On the other side of the country, Captain Henderson was working on his

1 Capt William Yolland, *An Account of the Measurement of the Lough Foyle Base in Ireland*, Ordnance Survey, 1847. His account of 'the various methods of computation' is at pp 144-151 and Appendices XII and XIII.

2 David L Walker, *A Fresh Look at the initial Ordnance triangulation etc*, *Sheetlines* 117, April 2020, 9-22

3 The writer's assistance from *Projections and Origins collected writings of Brian Adams* (CCS 2006) and *The Ordnance Survey in the Nineteenth Century* by Richard Oliver (CCS 2014) is too broad to footnote adequately.

triangulation of the Irish Sea, and in 1836 he provided positions for Criffell and Bengairn that were calculated on Airy's figure of the earth.

When Captain FW Beechey RN, an experienced hydrographer, sought a proper explanation of the differences between Henderson's figures and those published in 1811, Captain Robe RE had to respond that '*he must not communicate Col Colby's formula*' for the length of a degree and the ellipticity of the earth.⁴

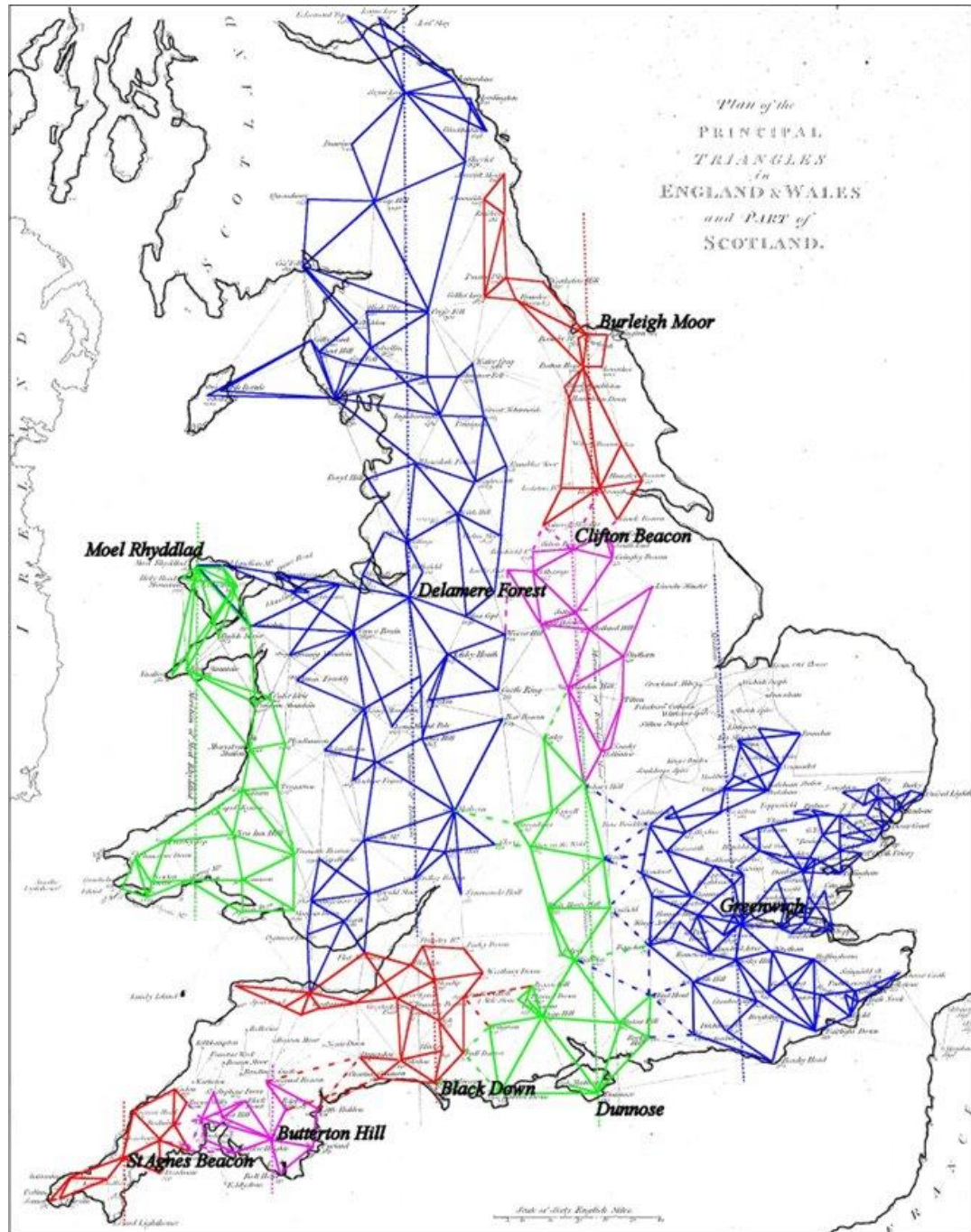


Figure 1. The initial triangulation as completed by 1811 (Faden Volume III Plate I)

⁴ David L Walker, *The Ordnance Survey and Airy's figure of the earth*, Sheetlines 119, December 2020, 6-17.

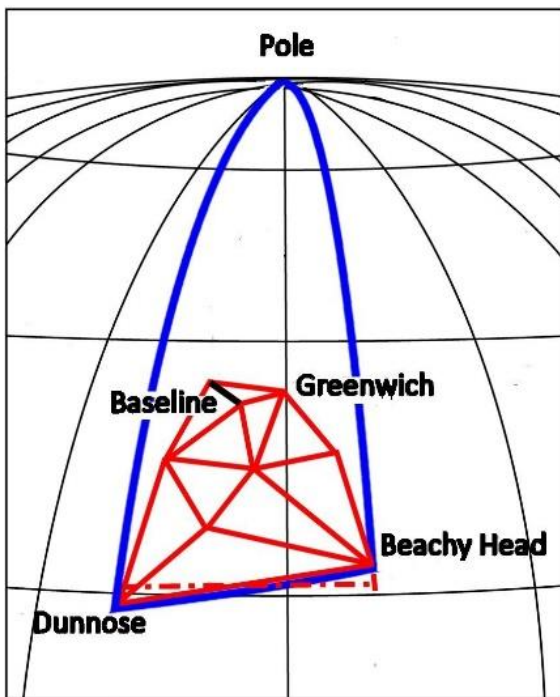
Perhaps it was Beechey's challenge that led Colby in 1841 to seek advice from Airy, by then Astronomer Royal, on the calculation of latitude and longitude based on a unified spheroidal model of the earth, instead of the separate meridians used previously. Airy tested his model against a circuit of the Scottish triangulation, largely observed by Colby, and he later collaborated with Yolland's mathematical team, enabling them first to test and then in 1847 to publish their slightly different version of Airy's spheroidal calculation.

As this article mentions Thomas Colby's failings (like William Mudge's) in not taking account of very relevant external advice, it is only fair to highlight his tenacity in the field, his endurance in dealing with the Treasury and his loyal support of William Yolland.

Yolland's indictment of the past, published in 1847

It was immediately after Colby's retirement in 1847 that Yolland's 'exhibit' of the various methods of trigonometrical computation, including his new model, was published within his account of Lough Foyle baseline. This timing avoided intervention by either Thomas Colby or his successor, Lewis Hall, but it unfortunately may have reduced attention to Yolland's spheroidal model, which was crucial to the future of the trigonometrical survey.

Remarkably, this 'exhibit' included a stringent critique of parts of the previous Ordnance triangulation. After a silence necessary for as long as he reported to Colby, Yolland's promptness in 1847 in publishing his account of previous mistakes perhaps reflected his built-up frustration as well as a wish to clear the slate for the future. To understand this critique, it is useful first to note how the length of a degree of longitude was estimated in 1795.



Having measured the distance between Dunnose and Beachy Head by triangulation, William Mudge and Isaac Dalby estimated the difference in longitude between these points from observations of the pole star, as in *figure 2*. But others expressed doubts over this approach.

John Playfair in 1798 warned that irregularities of up to 10 or 12 seconds in the gravitational attraction on a plumbline could arise from variations in underground strata as well as the surface terrain, and Captain Kater in 1828, discussing unavoidable inaccuracies in making observations of the pole star, concluded such observations 'for the purpose of determining the length of a perpendicular degree in our latitude, are wholly unworthy of credit.'⁵

Figure 2 Illustration of Longitude calculation based on Phil Trans Roy Soc 1795 Tab XLV figure 3

⁵ David L Walker, *The Ordnance Survey and Airy's figure of the earth*, *Sheetlines* 119, December 2020, 6-17.

Yolland's crucial criticism in 1847 was that the calculations for the trigonometrical survey had been made to depend entirely on elements derived from the work itself. No attempt had been made to combine these with the results of similar operations in other countries to determine a figure of the earth.

Moreover, tables of the lengths of degrees on and perpendicular to the meridian were calculated for all other latitudes from the number of fathoms in a degree on the meridian only 'for a certain latitude' and from the number of fathoms in a degree perpendicular to the meridian only 'for a certain latitude'.

Yolland's culminating indictment of the past was that '*Unfortunately for the accuracy which might reasonably be expected from the results furnished by a Trigonometrical Survey, executed with very superior instruments and great skill, the length of a degree perpendicular to the meridian was mainly determined by the reciprocal Azimuthal observations made at Dunnose and Beachy Head stations.*' He explained that the more important errors resulted from inaccuracy in the Azimuthal bearings [by reference to the pole star] and/or gravitational deflection of either or both of the plumb lines required. Also, instead of using all the observations of Polaris, as only some had been selected, '*The principle of selection in this case had probably done more damage than in any other.*'

His devastating conclusion was that all Longitudes calculated by the Survey [since 1795] '*being dependant [sic] on the length of a degree perpendicular to the meridian largely in error, are in defect about 1/300 part of their distance East or West of Greenwich.*'

Yolland seizes the initiative in the 1840s

Seymour's 'official' history of the Ordnance states that '*Clarke completed the work begun by Yolland, perfecting the methods that were used and directing the computations. The adjusted geodetic network was known as the Principal Triangulation, a term taken from Clarke's eight-hundred-page quarto volume ... published in 1858.*' This tends to create the misleading impression that after 1858 the topographical survey was based on Clarke's adjusted network, although it is qualified by the statement that '*The work of improving and strengthening the primary triangulation continued under Yolland during the 1840s and was eventually completed in 1852.*'

The spheroidal calculation model published by Yolland has been explained in several different sources and apparently it was slightly modified over subsequent years. The model as it was used in 1847 and recorded in TNA OS 2/647 is analysed in the Appendix and is shown there in figure 18 for the calculation of the latitude and longitude of Ben Lawers from Hartfell.

This model was put to use from about 1845 for the several stages of calculation that preceded the publication of Ordnance maps and plans, including:

1. re-calculation of the latitudes and longitudes previously adopted for the principle [sic] points of the triangulation, using updated observations and on the basis of Airy's figure of the earth;

2. calculation on the same basis of other county origins, and of secondary points, including coastal points for the Admiralty Hydrographer⁶ and for astronomical observatories; and
3. calculation of the latitudes and longitudes of Ordnance sheet corners.

Whereas these latitudes and longitudes were calculated on the spheroidal model of the earth, local trig points, also recorded in the volumes catalogued in TNA section OS 2, continued to be calculated by plane trigonometry and projected on the rectangular spheroidal or Cassini co-ordinates used in previous years.

Much of this work survives in the national archives. However, this writer's hope of finding every step in this process was frustrated, partly by the random loss of files resulting from the wartime bombing of the records at Southampton, but more by the selection of records for preservation in slices chosen 'horizontally' rather than 'vertically'. For example, voluminous records have been kept of the calculation of sheet corners, but no-one seems to have considered the preservation of a sample of field calculation books.

Nevertheless, the catalogue index to the National Archives at Kew is magnificent. It brings together that which is available and, importantly, indicates what is not available. This article also draws upon the British Library, the UKHO archive at Taunton and the RGO records from Greenwich now held in Cambridge. It illustrates only a small proportion of the work managed by Yolland in the 1840s, and perhaps others will be ready to remedy this writer's mistakes and ignorance of other sources.

The Cart changes place with the Horse

For fifty years the Ordnance first calculated the position of their trigonometrical stations in 'rectangular spheroidal' or 'Cassini' co-ordinates on great circles of the earth, and from these they afterwards calculated and tabulated their latitudes and longitudes. But this order was reversed around 1845. Before and after, the procedures compare as follows:

Before about 1845

1. Triangulation from south to north related to about nine meridians on nine origins.
2. Positions calculated at vertices of plane triangles on segments of nine irregular polyhedra.
3. Lengths of degrees of latitude calculated entirely from observations made in 1795.
4. Positions calculated as Cassini co-ordinates,
- and 5. later converted to latitudes and longitudes.
6. Six inch topography adopted Cassini sheetlines.
7. No indication of latitude and longitude.

After about 1845

1. Triangulation over a single network related to the Salisbury and Lough Foyle baselines.
2. Positions calculated at vertices of spheroidal triangles on the surface of a single spheroid.
3. Lengths of degrees of latitude and longitude calculated from Airy's figure of the earth.
4. Positions calculated as latitudes and longitudes,
- and 5. then converted to Cassini co-ordinates.
6. Six inch topography adopted Cassini sheetlines.
7. Sheetline graticule shows latitude and longitude.

⁶ UKHO, LP1857 Y contains Yolland's letters; those from the Admiralty Hydrographer are in UKHO, LB 10 to LB 14.

Yolland's Principle [sic] Points

The completion in 1852 of Yolland's work '*of improving and strengthening the primary triangulation*' was marked by his removal to Ireland. Fittingly, it was also marked by the exhibition of his updated triangulation of Great Britain and Ireland (figure 5) at the Great Exhibition of 1851.

So far as we know, only two (slightly different) copies of this diagram survive, in the British Library and the National Library of Scotland.⁷ Perhaps because of its similar title, it has more than once been confused with the more numerous copies of AR Clarke's diagram of his Principal Triangulation published in 1858.

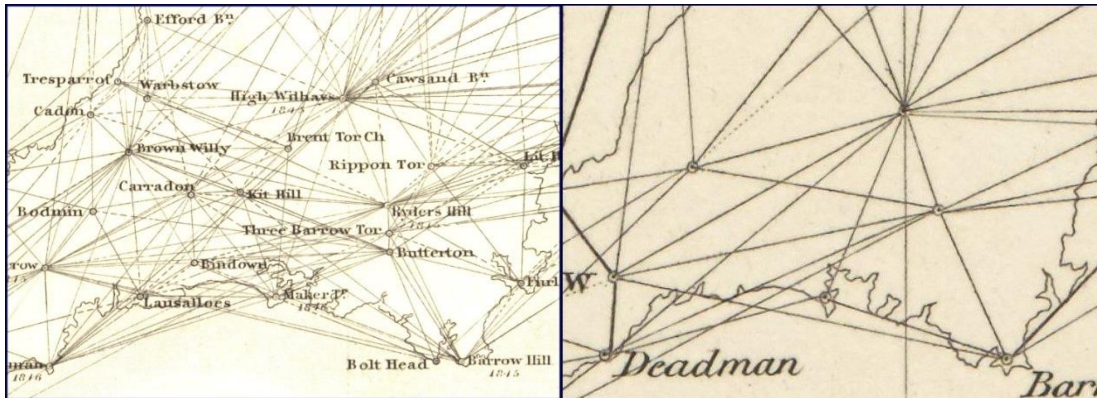


Figure 3 (left). Part of Yolland's triangulation diagram 1851.

Figure 4 (rt). Part of Clarke's triangulation diagram 1858.

In 1858, Clarke's refinement of Yolland's triangulation combined 'all' observed bearings using least squares analysis to satisfy all geometrical relations of the figure, although he chose to exclude bearings from a number of stations in some parts of the country as shown in figures 3 and 4.

In TNA OS 2/647 points included in the 1851 diagram were at first described as 'principle points', accidentally providing a distinction from AR Clarke's Principal Triangulation published in 1858. TNA OS 2/647 includes pages of calculations to test alternative models of spheroidal trigonometry and to build reference tables for the application of Yolland's model. With TNA OS 2/648, it also includes re-calculation of the positions of many triangulation points. The random survival of the calculations of the positions of Yolland's 'principle points' arises from the random survival of the Ordnance records preserved in the National Archives (now more of these at Harrogate than at Kew). However, confirmation that the Ordnance continued for a century to rely upon the calculations initiated by Yolland's team, unaffected by Clarke's magnificent but frustrated exercise in least squares analysis, comes from TNA OS 2/616.

Overleaf : Figure 5 '*Diagram shewing the Principal Triangulation of the Ordnance Survey of Great Britain and Ireland*', previously undated, British Library, Maps 1105(15) and National Library of Scotland, Map.Area.C16(144).

⁷ David L Walker, *The Ordnance Triangulation shown at the Great Exhibition of 1851*, *Sheetlines* 129, April 2024, 42 and Item XIII in Figure 6 (extract of the Official catalogue) from the Smithsonian Institution's website.

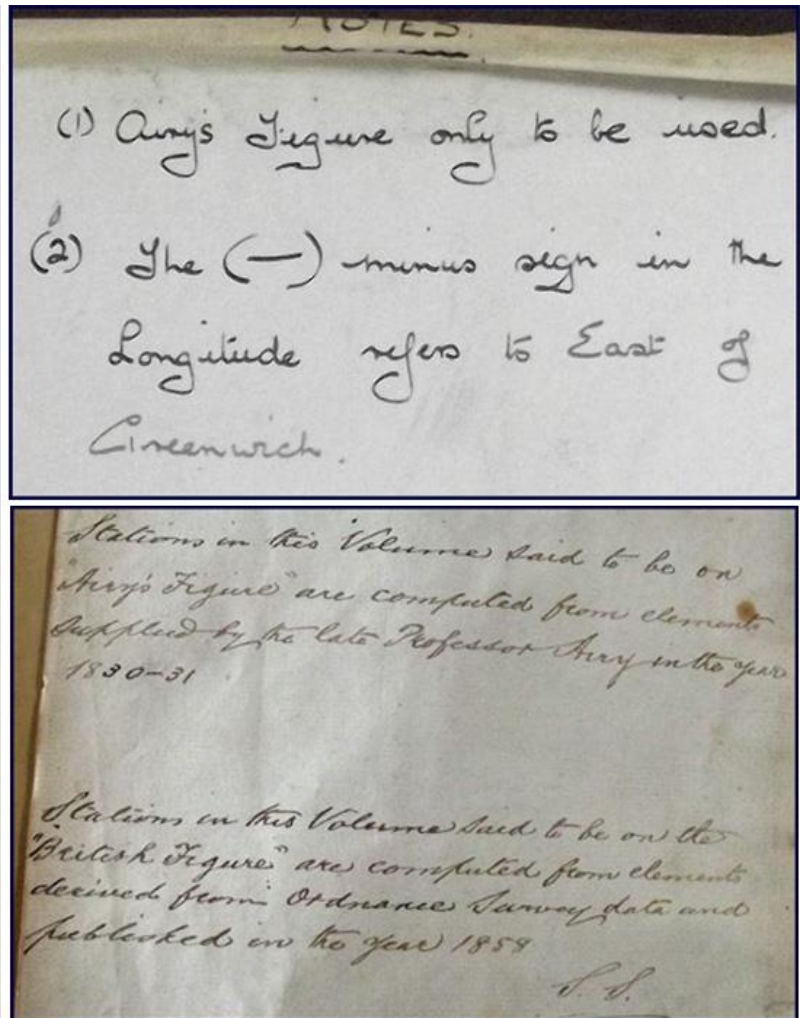
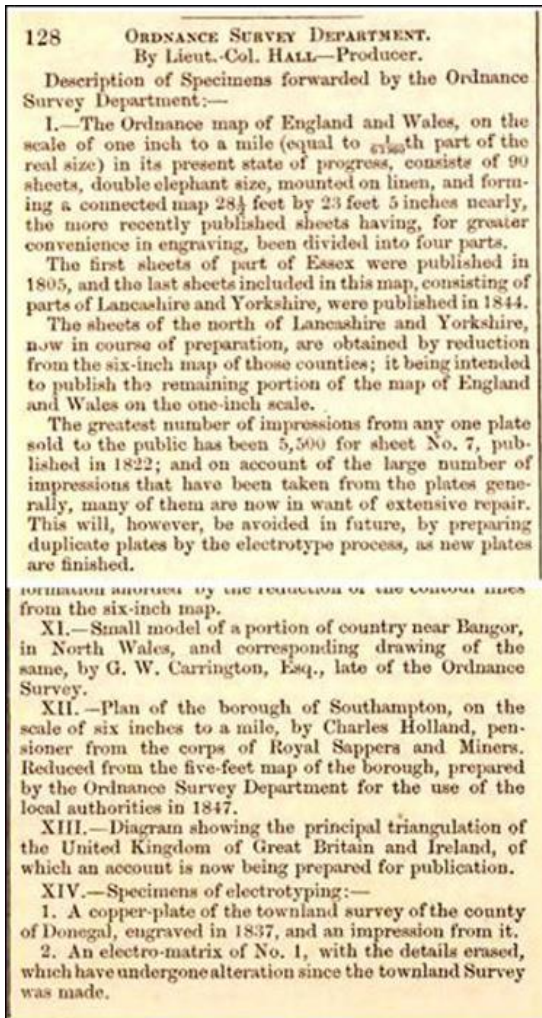


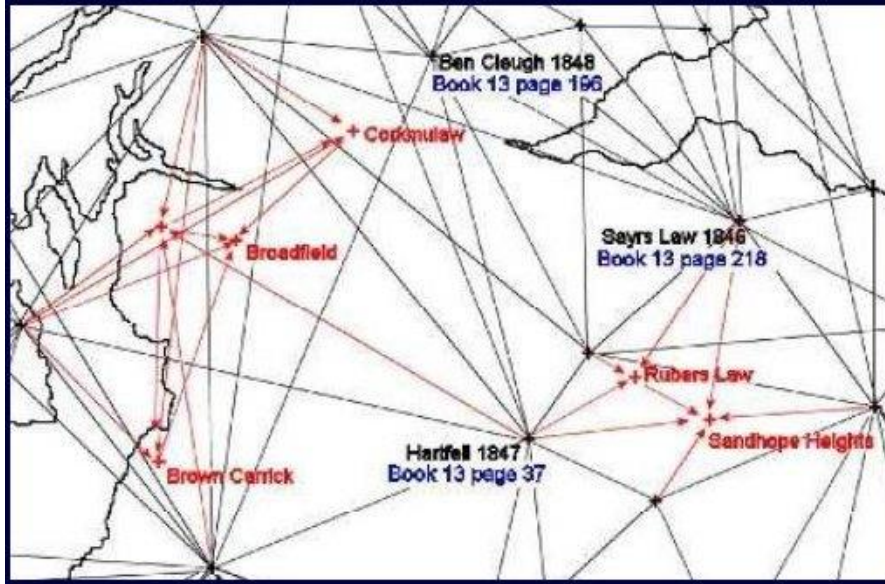
Figure 6 (left). Extract from Official descriptive and illustrated catalogue, Great Exhibition of 1851, Volume 1, 341-342. Figure 7 (top right). Note within TNA OS 2/616. Figure 8 (bottom right). Note attached to the cover of TNA OS 2/616.

As shown in figure 7, this well-worn listing of the latitude and longitude of Ordnance Survey stations includes one list calculated by Yolland and his successors from Airy's figure of the earth published in 1830, and another list calculated from AR Clarke's adjusted figure completed (too late) in 1858. The note inside the cover (figure 8) presumably reflected a disinclination to recalculate and adjust the use of existing reference tables already based on Airy's figure.

County Origins and Secondary Points

It is difficult to find any logical order within the 'Projection Books' in OS 2 and the order of their contents mainly seems to reflect the priority of the work as it came in, and the persons to whom it was allocated. The catalogue description for TNA OS 2/648 (Projection Book Number 16) is particularly variegated, covering the Latitudes and Longitudes of six-inch sheet corners of ten counties of England and Scotland; Latitudes and Longitudes of the Minor Points of England; Latitudes and Longitudes of the Initial Points of Scotland; Latitudes and longitudes of one-inch sheets in England and Scotland; and Co-ordinates of the Principal Points of Scotland.

To find the composition of the Minor Points of England we still need someone to work through this book (after taking care to order it from Harrogate a week beforehand). The Initial Points (or County Origins) of the six inch plans are more familiar, from Brian Adams' *Projections and Origins*.



Unlike OS 2/647, latitudes and longitudes of some initial points in southern Scotland were calculated from four or five 'principle' points and simply averaged. This is shown at Rubers Law, for example, in OS 2/648 page 244. Although averaging was to the distaste of one later commentator, it was presumably felt not unreasonable to average

results already within a range of a few thousandths of a second, and these figures survived within Scottish county origins.

Figure 9 (above). Scottish county origins calculated in TNA OS 2/648 (Projection Book No 16 dated 1856)

Latitudes and Longitudes of Sheet Corners

Their correspondence demonstrates that Captain Yolland, as he then was, enjoyed a very cordial relationship with George Airy, the Astronomer Royal.⁸ However, 'it had long been urged as a complaint' by Airy and others that latitudes and longitudes were absent from Ordnance maps. Becoming confident after 1845 of the methodology to put this right, Yolland tackled the margins of the six inch series, starting with Yorkshire and Lancashire in England and Wigtownshire in Scotland. Wigtownshire provides a convenient example as the documents are clearly dated in 1846 (and it is a much smaller county).

The Knock of Luce provided the county origin and its central meridian. After the county boundary had been clearly identified, on the coast and inland, the layout of the 6 inch sheets would have been decided to best advantage, and slightly adjusted to place Knock of Luce at a round number of feet from its own sheet boundaries, as in figures 10 and 11.

The latitude and longitude of each sheet corner was calculated in turn, employing Yolland's methodology, using its distance and azimuth from Knock of Luce calculated by Pythagoras and tangent respectively. Thus, for example, the relative co-ordinates of the NW corner of sheet 11 are 40,540 ft north and 71,760 ft west respectively, equal to a distance of 82,419.6 ft and azimuth of

⁸ Correspondence with Col. Yolland, *Papers of George Airy*, RGO 6/416 & 417, Cambridge University Library.

60 degrees 32 min 10.4 secs from Knock of Luce, and these figures were used in the following calculation, shown in *figure 12*. The outcome for this particular sheet corner is shown in *figure 13*.

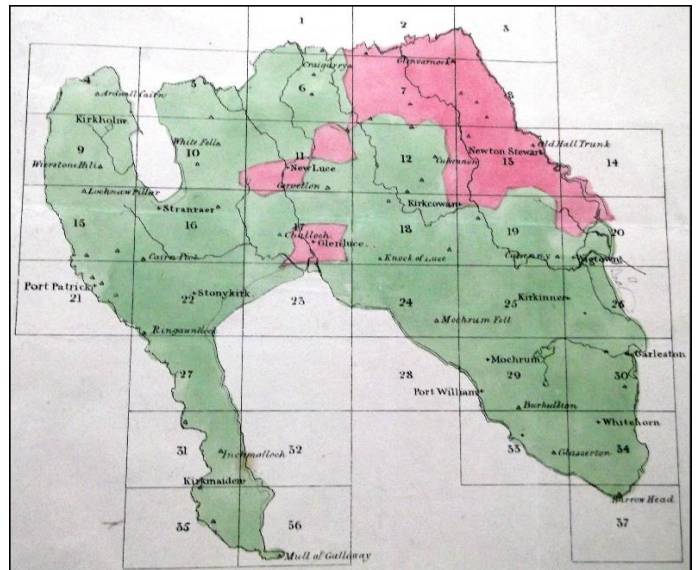
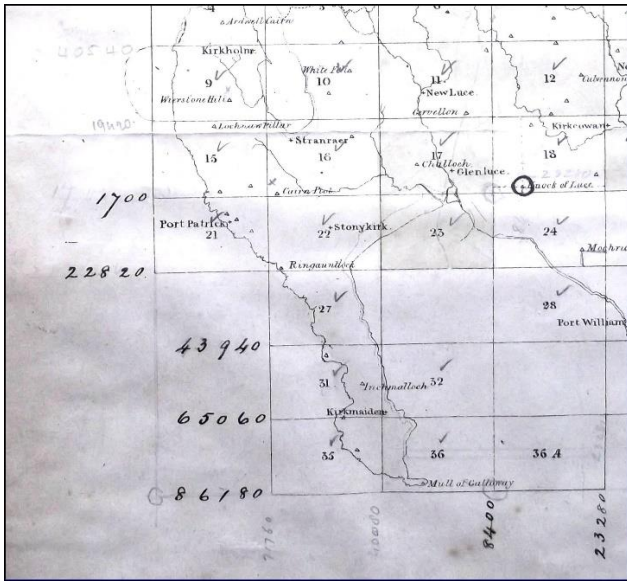


Figure 10 (left). [part of] Index (undated) showing the state of the Ordnance Survey in Wigtownshire, from OS 2/683. Sheet 36A, if used, would have contained Great Scare Rock (shown on NLS website Admiralty Chart 1971 publ 1846).

Figure 11(right). Index (same title but dated 5 Dec 1846), from OS 2/683 County Sheet Book Wigtownshire 1845-1846.

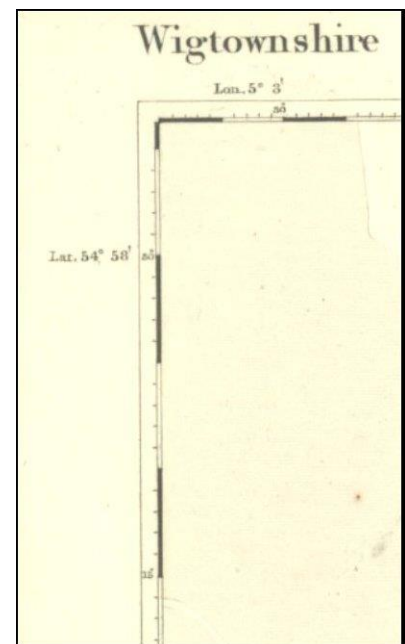
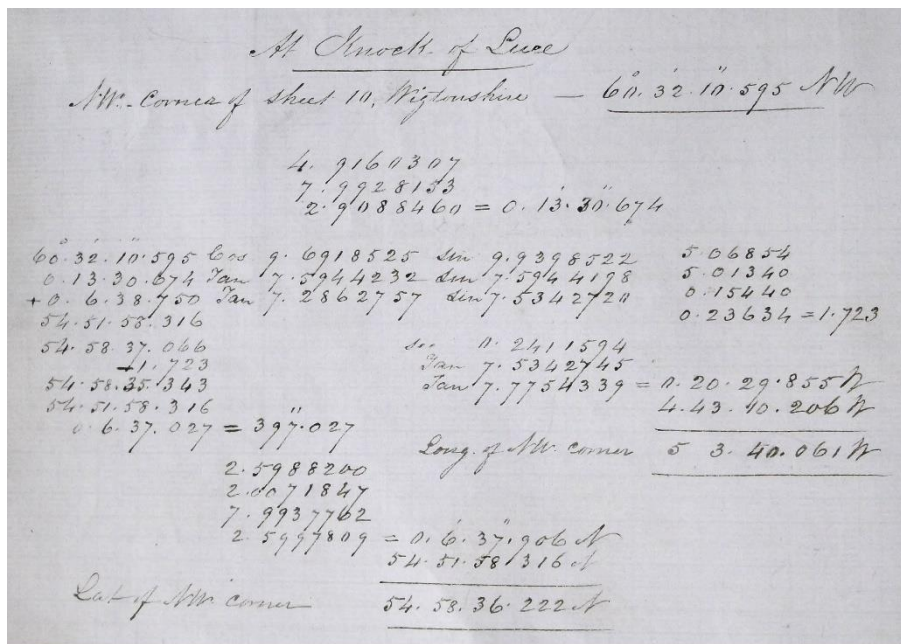


Figure 12 (left). Calculation sheet for Wigtownshire Sheet 10 NW corner. Figure 13 (right). NW corner Sheet 10 (from NLS)

Co-ordinates of Local Points

Under the revised procedure adopted after 1845, the positions of principal and all other points to be used to define local trig points were converted from latitude and longitude to 'rectangular spheroidal' or 'Cassini' co-ordinates. This

procedure used the extensive conversion tables that were later published in *Methods and Processes*⁹ and was documented eg in OS 2/648.

C^o of Edinburgh
Sheet lines from 3500 ft. to 17000 ft. and 16700 ft. to 17000 ft. West off

SHEET N^o 1

Distance on and from the Meridian of Edinburgh Cast. Distance from Sheet Lines

Names of Stations	Stations from which obtained	Page	On Meridian	From Meridian	From South Line	From East Line
			3500 0	16700 0		
<i>Cramond Island</i>	<i>Cramond Island</i>	2 154	7569 18	50457 38		
<i>Muir House tower</i>	<i>Muir House tower</i>	157	7569 9	50457 3		
<i>Cramond Ch. tower</i>	<i>Cramond Ch. tower</i>	153	7569 1	50456 0		
<i>Flag staff</i>	<i>Flag staff</i>	154	7569 0	50457 1	10769 0	50457 1
<i>Muir House tower</i>	<i>Muir House tower</i>	2 157	6756 38	17251 38		
<i>Edinburgh Observatory</i>	<i>Edinburgh Observatory</i>	154	6756 6	17252 1		
<i>Cramond Ground</i>	<i>Cramond Ground</i>	155	6756 4	17251 1		
<i>in tree</i>	<i>in tree</i>	154	6756 1	17251 5	5276 1	5271
<i>Cramond Island</i>	<i>Cramond Island</i>	2 154	1896 48	506219 48		
<i>Muir House</i>	<i>Muir House</i>	154	1896 7	506219 4		
<i>Edinburgh Observatory</i>	<i>Edinburgh Observatory</i>	157	1896 3	506219 4		
<i>Bridge</i>	<i>Bridge</i>	154	1896 1	506220 2	5276 1	5271
<i>Edinburgh Castle</i>	<i>Edinburgh Castle</i>	2 154	1896 38	18500 38		
<i>Arthur's Seat</i>	<i>Arthur's Seat</i>	154	1896 5	18500 2		
<i>Cramond Island</i>	<i>Cramond Island</i>	154	1896 9	18500 7		
<i>Carmethy Cairn</i>	<i>Carmethy Cairn</i>	154	1896 1	18500 1		
<i>Observatory</i>	<i>Observatory</i>	154	1896 8	18500 8	19758 8	18520

This meant no change was required in the plotting of local trig points for the topographical survey. These were calculated as if on a plane surface, and plotted on rectangular spheroidal or Cassini sheetlines. Field books showing how local observations were calculated as the sides and angles of local triangles are hard to find, but a final stage is indicated in this example for Cramond Island, that some readers will have

observed on the approach to Edinburgh Airport (Figure 14 (left)). From TNA OS 2/674 County Sheet Book Edinburghshire.

A lower part of the page shown in figure 14 lists the additional points shown in figure 15, and the points from which these were measured are shown in figure 16. But this routine is disappointing in revealing only one side of the lowest order triangles of the trigonometrical survey. If these depended on narrow angles (which is unclear), this may have been because Edinburghshire Sheet 1 made an isolated start within its part of the topographical survey.

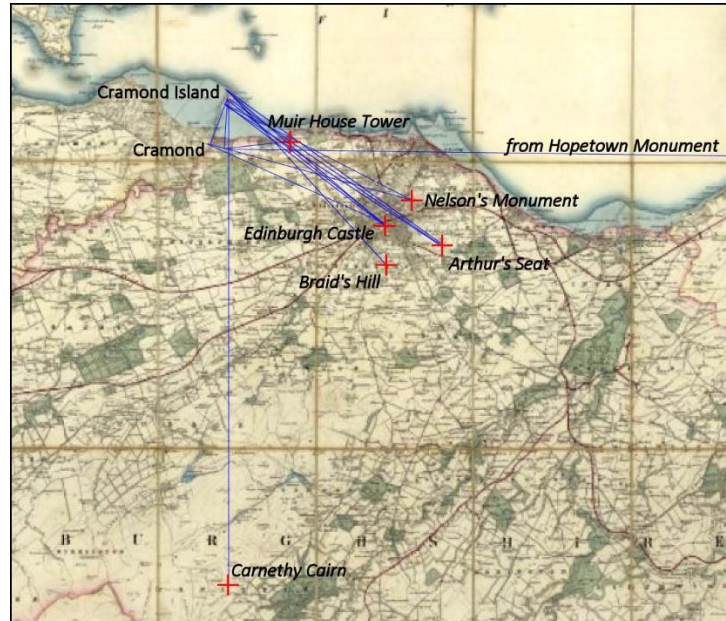
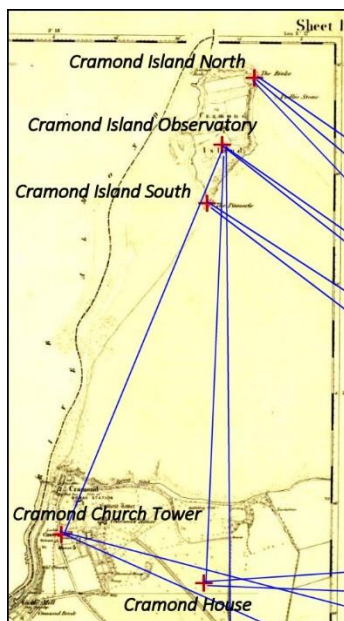


Figure 15 (far left). Cramond Island trig points.
Figure 16 (left). Trig points from which Cramond Island was observed.

⁹ Ordnance Survey, *Account of the Methods and Processes adopted for the production of the maps of the Ordnance Survey of the United Kingdom*, 1875. This is informative, but confidence is weakened by statements that 'all the north and south edges of the [six inch] sheets are by construction parallel to the meridian of the county' and 'for calculation of the triangulation all observed bearings were combined using least squares'.

Conclusions and Acknowledgements

Alongside the work outlined above, William Yolland fostered the Survey's relations with the Astronomer Royal, George Airy, and the Admiralty Hydrographer, Francis Beaufort. Also notable was his publication of past and current work, including that covered by the Appendix.

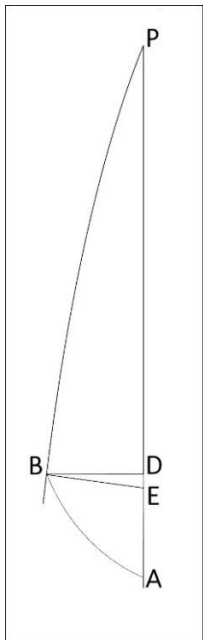
In 1854 Major Yolland became an inspector of railways and from 1877 was personally effective as Chief Inspector of Railways until his death in 1885. His distinguished career is recorded in the Dictionary of National Biography, unlike that of his nemesis, Lewis Hall.

The writer warmly thanks Rob Wheeler, Chris Fleet at the NLS, Anne Taylor at the CUL, and the staff of The National Archives for their assistance and encouragement.

Appendix

Yolland's method for point to point calculation of latitudes and longitude

In 1847 Yolland explained this new procedure in the six steps summarised below.¹⁰ Early applications of the procedure appear, for example, in TNA OS 2/647, and its use from Hartfell in Dumfriesshire to Ben Lawers in Perthshire is illustrated in figure 18.



In Yolland's explanation and in figure 17, P is the earth's pole and A a point of known latitude and longitude on a longitudinal meridian PA. The aim is to determine the latitude and longitude of point B on the meridian PB, having observed the azimuthal bearing PAB of B from A, and having calculated the distance AB by triangulation from a measured baseline.

All triangles are spherical triangles unless otherwise stated. BD is an arc of the great circle through B that intersects the meridian PA at 90 degrees at D whereas BE is an arc of the parallel of latitude through B that intersects the meridian PA at E.

Figure 17 Sector of the earth's spheroid as adopted for Yolland's calculation procedure.

1. Step 1 was to convert AB to seconds of arc using any approximate radius of the earth and solve ABD as a spherical triangle right-angled at D to calculate arcs AD and DB.

2. Then the co-latitude of point D = arc PA (co-latitude of A) less arc AD.

3. Then solve the right-angled spherical triangle PDB to find arc PB, the co-latitude of B or E, and the difference of longitude BPD on the approximate sphere.

4. Take AE in seconds (in latitudes A and B), convert to feet using the approximate radius, and convert feet to seconds on the meridian using Yolland's Table II Appendix XIII for mean latitude of A and B (and hence true difference of latitude on the assumed figure of the earth).

5. Compute BE in seconds ($= P \cdot \sin DP$), convert this to feet with assumed approximate radius, and to seconds of longitude using spheroidal radius of parallel for latitude B (or E).

6. Find $PAB + PBA$ from $\tan((PAB + PBA)/2) = \cos((PA - PB)/2) \times \cot(APB/2) / \cos((PA + PB)/2)$ and the azimuth of A from B by subtracting the angle PAB. This procedure was empirical, the approximate radius being introduced after trying out variations of Airy's suggested procedure until finding one (suggested by one of his team) that returned to the same point after completing a circuit of the triangulation.

Figure 18 shows their calculation of the position of Ben Lawers relative to Hart Fell, annotated in red by the present writer to match the six steps. His notes in figure 19 nearly,

¹⁰ Yolland, Wm., *Account of the measurement of Lough Foyle Base, Ordnance Survey, 1847*, 148.

but not entirely, explain the figures in figure 18. From later records it emerges that Yolland and others could not resist making minor adjustments. Nevertheless, the writer hopes that these notes may provide a useful starting point for anyone wishing to take this further.

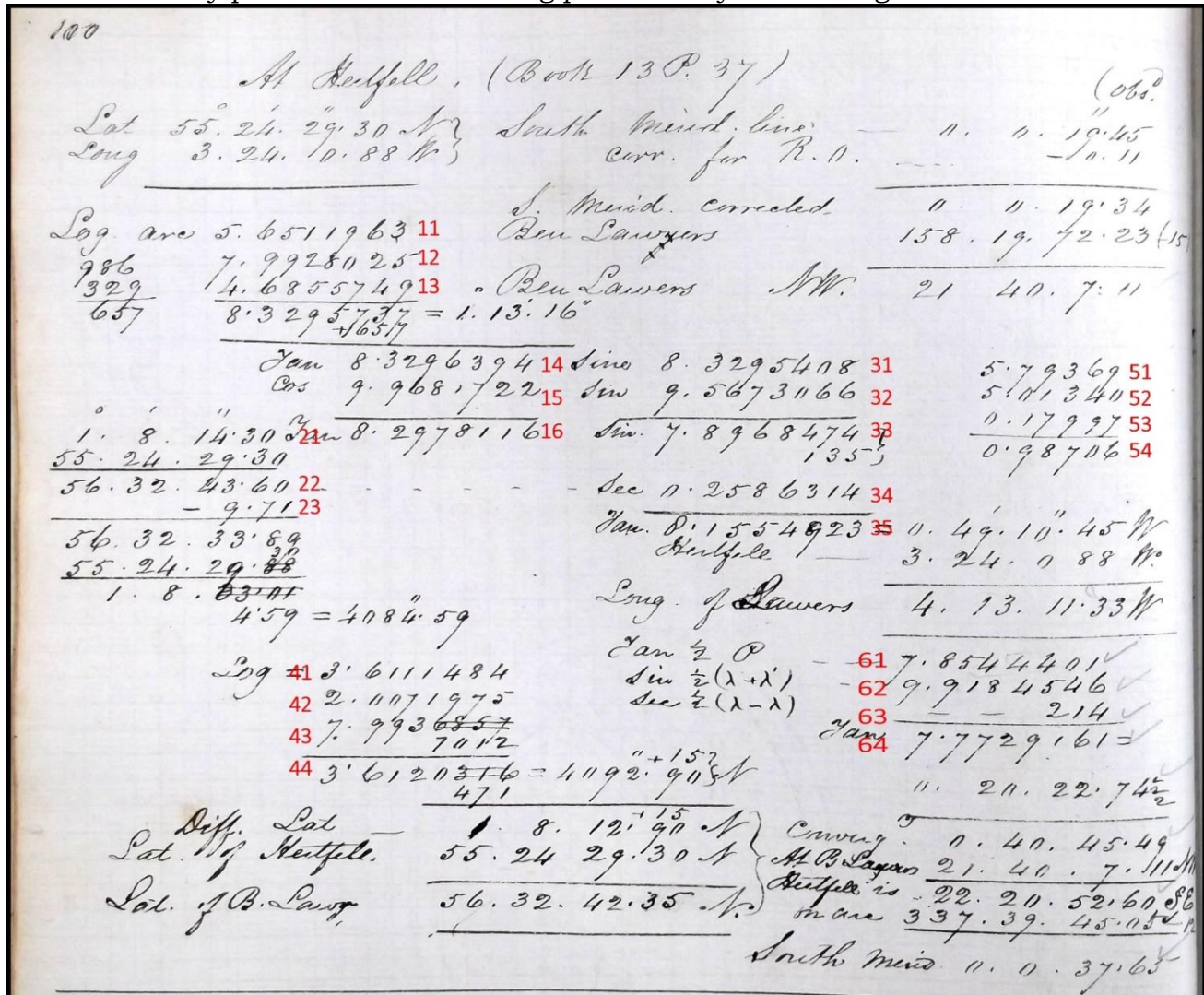


Figure 18. Hartfell to Ben Lawers from TNA OS 2/647, 100.

11	log arc AB in ft	447,915.7	Hart Fell to Ben Lawers	31	log sin arc AB*	1°.13'.25.54
12	ft to secs perp to meridian, Lough Foyle, Table II.			32	log sin angle BAD*	azimuth 21°.40'. 7.11"
13	log arc AB in seconds			33	log sin arc BD*	
14	log tan arc AB*		*actually log +10	34	log sec	initial latitude of B
15	log cos angle DAB*		azimuth A to B	35	log tan BPD*	0°.49'.10.45"
16	1°.08'.14.30"		$\tan \text{arc} DB = \tan \text{arc} AB \cdot \cos \text{angle} BAD$	51	step 5	not yet matched
21	AD (initial estimate)		1°.08' 14.30	52	step 5 (cont'd)	not yet matched
22	initial latitude of B (Ben Lawers) on approx radius			53	log tan initial latitude of B (Ben Lawers)	
23	DE (in seconds)		DE from line 54	54	log DE in seconds	to line 23
41	AE initial est	4084.59"		61	=(line 35)/2 tangent	0°.24'.35.2"
42	complement/line 12, excludes approximate radius.			62	sine	34°.01'.24"
43	ft to secs on the meridian, Lough Foyle, Table II.			63	secant	0°.34'.7"
44	AE corrected	4093.05"		64	tangent	0°.20'.22.746"

Figure 19. Notes on Figure 18 (Hartfell to Ben Lawers)