Brian Adams (breviter) Redux

Michael Spencer

Introduction

In 2006 the Society published *Projections and Origins: collected writings of Brian Adams,* edited by Roger Hellyer and Chris Higley. Adams (1924 – 2005) was a hydrographic cartographer with a particular interest in the mathematical underpinnings of mapmaking, and he was a frequent contributor to *Sheetlines* in this general field, in which "he was not just one expert, he was the expert" (from the Preface to *Projections and Origins*, unsigned but presumably written by one of the editors thereof). He also contributed an Appendix to Roger Hellyer, *The 'ten-mile' maps of the Ordnance Survey*, published by the Society in 1992; this Appendix is reproduced in *Projections and Origins.*

These writings of Adams were designed of course to clarify his somewhat difficult professional ideas to those members of the Society who were interested in but not entirely comfortable with them. None the less, it seems to me that he omitted to explain in any degree at all two concepts mentioned in the course of the Appendix to *Hellyer*, in the sentence:

> "The projections of all Ordnance Survey maps of Great Britain, of regular series commenced after 1830 and on scales of 1:633,600 and larger, are calculated on the Airy spheroid (Airy's figure of the earth) defined in terms of the foot of Bar O_1."[1]

The concepts of the Airy spheroid and the foot of Bar O_1 are not defined anywhere by Adams, no doubt because these are fundamental ideas in the field of mathematical cartography and were as familiar to Adams as the concepts of Boolean logic used to be to me. It is the purpose of this paper to put some flesh on those bones; and, in particular, the Lemma to the discussion of the "Bar of O_1" raises an important question that does not seem to have previously been addressed.

Some further questions arising from Adams' work will be examined in a forthcoming article.

The Airy Spheroid

The Ancient Greeks were aware that the Earth is a spherical body rotating slowly about its polar axis, and they developed map projections designed to allow the depiction of the surface of such a body on a flat sheet of paper. It took more than two thousand years for mathematical physics to develop to the point where it could be understood why such a large body was necessarily spherical, and why and by how much its slow rotation distorted it so that the equatorial diameter was larger than the polar diameter. A sphere distorted in such a way is called a *spheroid*. The nineteenth-century science of *geodesy* was able to determine the real size of the spheroid and to encase it in a mathematical straitjacket that allowed the conversion of geographical (spheroidal) co-ordinates of points on its surface to rectangular (map) coordinates of points on paper. Any formula to do this is called a *projection*. Spheroidal co-ordinates are called *latitude* and *longitude*; map co-ordinates are called *eastings* and *northings*.

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[1] Roger Hellyer, op. cit, (1992), p. 176; Roger Hellyer and Chris Higley (eds), op.cit. (2006), 5

George Biddell Airy (1801-92, Astronomer Royal 1835-81) was an early geodesist. In 1830, he calculated the lengths of the polar radius and equatorial radius of the earth using measurements taken in the UK and France. He gave these dimensions[2] as:

a – the equatorial radius = 20,923,713 feet
b – the polar radius = 20,853,810 feet

To bring both the Imperial and the metric measures into a unified document, a conversion between feet and metres was required. This had been obtained in 1817 by Capt. Henry Kater, comparing the Imperial standard used by Airy with a platinum copy of the Archived Metre whose length had been determined in 1799 by François Arago, the leading French geodesist. Kater found that

Foot/metre = 0.3048007491

At the time Airy's results were taken as state-of-the-art. Although his measurements were superseded by more accurate radius figures (such as those used for WGS84, the modern world geodetic system used by the Global Positioning System), his *Airy spheroid* is still used by the Ordnance Survey for the mapping of Great Britain because it better fits the local sea level (about 80 cm below world average). Those parts of the Airy spheroid that do not lie under Great Britain are of no value and have never been used.

The spheroid is also called the *ellipsoid*: the two terms are interchangeable, and choice of which to employ seems to be related to date. When the Ordnance Survey was young, the term *spheroid* was more usual, and so the phrase *the Airy spheroid* comes naturally to mind. In modern writing, the term *ellipsoid* seems to be preferred. Personally, I would tend to deprecate this usage, because it tends to hide the fact that even with the distortion, the earth is still fundamentally spherical. On the Airy spheroid, using the values for the diameters rather than the radii, because this simplifies the mental arithmetic, in round numbers the difference between the equatorial and the polar diameters is 140 thousand feet, while the equatorial diameter is 42 million feet: the ratio between these numbers, and so the magnitude of the distortion, technically called the *flattening*, is 0.3 per cent.

Lemma: Mapmaking before Airy

There was plenty of mapmaking in Britain before 1830. One thinks in particular of Saxton (1570s), Speed (1611) and Ogilby (1670) in England, Roy (1750s) in Scotland, and of course the early Ordnance Survey (by 1805) in Essex and Kent. All these people published maps for public consumption, and one has to ask, what did they think they were maps of and what projections did they use? In the particular case of the early Ordnance Survey, these maps were clearly *not* calculated on the 1830 Airy spheroid defined in terms of the foot of Bar O_1. So what did they use for their standard of length?

The foot of Bar 0_1

The practice of triangulation enables the relative positions of points on the surface of the earth to be established with remarkable accuracy, but to find the actual distances between points requires the use of an immutable standard of length to which

[2] GB Airy, Figure of the Earth (1830), in Encyclopedia Metropolitana, Vol. V, 165-239.

measurements can be referred. Four such standard bars are relevant to our story: Roy's 42-inch scale, the Shuckburgh scale and the wrought iron bars described as O_1 and O_2.

Major-General William Roy (1726-90) came to prominence after his success in mapping Scotland, completed in 1755, and was invited to lead the triangulation operations starting in 1784 to connect the positions of the observatories at Paris and Greenwich. His standard of reference was a 42-inch brass scale made in 1742, which he checked by microscopic observations against the National Standard held by the Royal Society. He used this to determine the length of a baseline he had set up on Hounslow Heath, to an precision of about 3 inches in 5 miles, or 1:100,000. This was a precision far exceeding any previous measurement, and Roy was recognised by the award of the Royal Society's prestigious Copley Medal in 1785.

The triangulation itself was delayed until 1787, awaiting the design and construction of an improved theodolite, and was completed in 1790. This theodolite was described by Sir Charles Close [3] as "the first in the world capable of detecting the spherical excess," that is the amount by which the angles of a triangle on a sphere or a spheroid exceed 180°. The calculations to convert the measured positions on the ground to rectangular co-ordinates for drawing the map were to some degree deficient. Seymour's *History* does nothing to reduce confusion here, stating [4] that Roy's calculations of latitude depended on Bouguer's spheroid (calculated in 1749 after observations made in Lapland, in France and in what is now Ecuador), and a few lines lower down that the calculations assumed that the surface of the earth was in fact plane, not spheroidal at all!

After Roy's death, the triangulation was extended (1791-1822) to cover the whole of Great Britain: this operation is known as the Principal Triangulation. The baseline on Hounslow Heath was re-measured, using 100-foot steel chains supplied by Jesse Ramsden. These were calibrated against his prismatic bar, which had in turn been compared carefully with the bar belonging to the Royal Society. When the calculations of the Principal Triangulation to determine the rectangular co-ordinates were to begin, it was felt that a new reference standard was required. In 1827 the instrument makers Troughton & Simms supplied two ten-foot bars of wrought iron, designated O_1 and O_2, each of dimensions 122.15 by 1.45 by 2.5 inches. They were brought to the temperature of 62°F, and dots were then engraved on them to mark the ten-foot standard, from which the length of one foot could be easily obtained – the "foot of bar O_1." The calculations were then carried out on the Airy spheroid, with the assumption that Airy's figures for the radii of the spheroid were in fact feet of O_1, which may have led Adams into his suggestion[5] that Airy based his results on such feet.

However, it is clear from Airy's own article in the Encylopedia Metropolitana that his reference was the Shuckburgh foot. Sir George Shuckburgh (1751 –1804) was a British politician, mathematician and astronomer. The Shuckburgh scale was a five-foot brass bar made by Troughton in 1796, originally for use in Shuckburgh's

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[3] CF Close, *The Early Years of the Ordnance Survey* (1926), p.15 (reprinted Newton Abbott: David & Charles, 1969).

[4] WA Seymour (ed), *A History of the Ordnance Survey* (Folkestone: Dawson, 1980), 36.

[5] In the extract from the Appendix to Hellyer, quoted at the start of this paper.

researches into metrology, engraved along its length in tenths of an inch. It was this Shuckburgh foot that Kater compared with the Archived Metre to find his conversion value.

Important conversion values

The Ordnance Survey made extensive comparisons between the O_1 bar and various other standard bars, of which the most important were:

- (made in 1742) Roy's Scale, a brass scale 42.8 inches long, divided by lines into inches and tenths of an inch;
- (1791) Ramsden's Prismatic Bar, a cast-iron bar 21 feet long of equilateral triangular section, 1.25 inches on each side, divided at 54°F into 40-inch parts marked off by dots engraved on brass pins let into the bar;
- (1796) the Shuckburgh scale: Edward Troughton's brass scale 66 inches long, divided by lines on silver into inches and tenths, and also divided into inches by dots on silver pins let into the brass.

From these comparisons, a conversion figure between each of these scales and the mean foot length of the O_1 bar was found, which became the standard of length for all activities of the Ordnance Survey until the advent of metric standards and the National Grid following the recommendations of the Davidson Committee[66] in 1938. (The use of the designations O_1 and O_2 may have simply been Troughton & Simms' codes to show that the bars were made for the Ordnance Survey; but they may also lead to confusion with tool steel of those grades, which were not available in the early nineteenth century. There is thus no need for doubt that the Ordnance Survey's bars O_1 and O_2 were made of wrought iron.)

For our present purposes, the ratio between the O_1 foot and the Shuckburgh foot is important, because it enables us to see how far the Ordnance Survey's use of the former led to a compromise of the actual size of the Airy spheroid. Any direct comparison between these two values is hidden in the hundreds of pages of mathematics available on the Internet, but fortunately both have been given in terms of the metre, from which their mutual ratio can be found. Thompson (1952)[77] gives both, and also gives a brief survey of the method of finding the ratio between the foot of O_1 and the metre, which involved comparison of the O_2 bar with the Standard Metre held in France, and then the comparison of the O_1 bar with O_2, followed by temperature corrections. In 1906 the method gave the result:

Foot of O_1 at 62°F = 0.304800756 metres.

Taking this together with Kater's result, we see that

Foot of O_1/foot of Shuckburgh = 0.304800756/0.3048007491 = 1.0000000023

or, the increase in the equatorial radius of the spheroid using the O_1 foot is insignificant (less than one foot in 20,923,713).

These days physical objects are no longer used to define the "truth": for example, the metre is now defined as the *length of the path travelled by light in a vacuum in 1/299792458 of a second.* This is just as well, because the present location of the Ordnance Survey bars is sadly not now known.

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[6] Seymour, op. cit., 257-266.

[7] Lieut.-Col. E.H. Thompson, RE: *The Ordnance Survey Foot/Metre Conversion Ratio* (1952), in *Empire Survey Review*, Vol.11, No. 84, 280-281.