

## A complex thing made simple with GeoGebra

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This article was inspired by Chris Sangwin's recent contribution on GeoGebra and geometrical functions in *MSOR Connections* [2], and, of course, motivated by Markus Hohenwarter's call for GeoGebra contributions for this issue. The author is grateful to both, as well as to the four students who agreed to have their work quoted here.

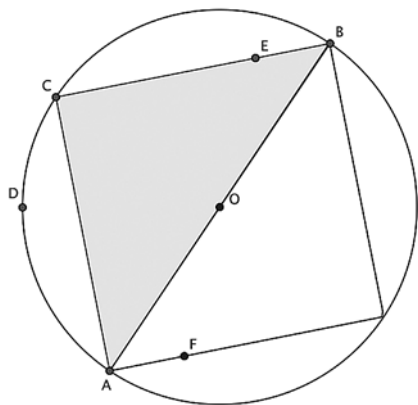
In his article, Sangwin applies a GeoGebra tool to illustrate Monge's theorem relating the intersections of certain tangents to circles. With this example he identifies a GeoGebra tool with a geometrical function. However, it was Sangwin's other example of a geometrical function (or GeoGebra tool), namely the extension of multiplication from the domain of real to that of complex numbers, which led me to ask: how can GeoGebra be used to visualise complex multiplication? Sangwin addressed the question of visualising  $z^2$ , given  $z$ , a point constrained to an arbitrary circle.

I wanted to consider the multiplication  $wz$ , where  $w$  is a point on the unit circle. My students had a reasonable background in coordinate geometry and complex numbers (from school), in vector and linear algebra (from university), as well as a few weeks' experience of Euclidean geometry with GeoGebra. (In Ireland, in recent years, the experience of formal geometry at school is rarely satisfying; one student encapsulated it as "having to learn off the fourteen theorems".) I anticipated using the lab-based GeoGebra session to introduce new features in GeoGebra ( **$x(A)$**  and  **$y(A)$**  for the coordinates of  $A$ , **Create new tool** for complex multiplication, and **Locus**). I expected that students would have little notion that the function  $z \mapsto wz$  had anything to do with rotation. To convince them of this was my main goal; a subsidiary goal (for later) was to link this complex multiplication to matrix multiplication.

In the lab-based session (23 February 2009), I presented the material in four stages:

- a. I gave the students a hard copy of the diagram (Fig 1), together with the given instructions.
- b. Then I moved the five points mentioned without revealing the structure behind them (see Figs 2, 3).
- c. I showed the students the Construction Protocol (see below), explaining the GeoGebra features new to them.
- d. Finally, I guided them through the construction of the GeoGebra worksheet, encouraging independent exploration.

The work of the (four) students was examined. Their rough notes represented three phases: the static situation, the dynamic picture, and the structure together with an emerging understanding.



Look at the diagram. Describe what you see.  
I am going to move each of the free objects in turn:  
E, D, C, B and A. Describe what is happening.  
Next consider the Construction Protocol ...  
Modify/revise any of your descriptions.  
Write down any questions you may have ...

Fig 1 – Hard copy of diagram given to students

Even in such a simple illustration, the static situation was described with rich variety. **Michael wrote:** We have a circle with centre O and points C, B, A on the circle ... There is a point E on the triangle ... There is a 2<sup>nd</sup> triangle where F is a point on the triangle. **Naomi wrote (but crossed out):** ... Mirror the triangle CBA to the other half of the circle. Place F on mirrored (symmetric) to the point E. **Ciara wrote:** A square inside a circle split into two triangles with a diagonal. One triangle is shaded red. E is a point on the shaded triangle. F is a point on the other triangle. D is a point on the circle. **Brendan wrote:** ... 2 triangles opposite can be seen, making up the measurements of a circle ...

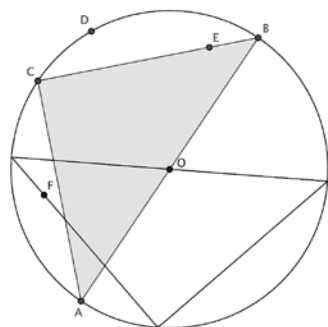


Fig 2 – Moving points within Geogebra

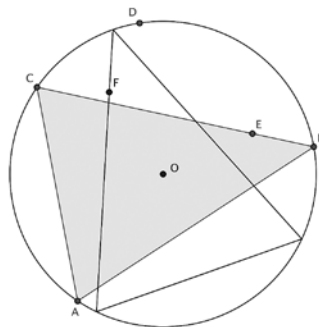


Fig 3 – Further movement of points within Geogebra

The varied, yet acute, observation continued in the description of the dynamic picture. **Michael:** Move E around the triangle ... F also moves on the corresponding triangle. Move the point D, which is constrained to the circle, the 2<sup>nd</sup> triangle then moves around about the circle. **Naomi:** When E is moved, F stays as the mirror image of E, so as E moved F also moves to be symmetric. When D moves, the mirrored triangle also moves. Why???? **Ciara:** E moves – so does F in the same way. D moves – so does the outlined triangle. C moves – so does [one] other point of the other triangle ... **Brendan:** When E is moved, F also moves. E moves around its triangle and so does F in its own triangle in opposite direction ...

The structure became clearer when the Construction Protocol was revealed. Here it is abbreviated:

1. Point O [at origin]
2. Circle c with centre O and radius 1
3. Points A, B, C on c
4. Triangle p [joining A, B, C]
5. Point E on p
6. Point D on c
7. Point F cMult[E,D] (the tool for complex multiplication)
8. Locus[F,E]

**Michael noted:** cMult[E,D] – point F, when D is moved this is the movement of F. Rotates round centre O, I think. **Naomi:** cMult[E,D] – As D moves, F moves similar to E; as E moves, F moves in symmetry. Locus[F,E] – GeoGebra draws the path F takes ... The similar triangle copies triangle CAB ... **Ciara:** 2 triangles remain congruent. **Brendan:** As D moves around the circle, the triangle opposite ACB also moves. As C ... moves, the size of the triangle ACB changes and the size of the opposite triangle also changes.

The insights achieved by the students, both individually and collectively, during a period of about 80 minutes, were significant. In general, the thinking was sharp, if, on occasions, its articulation might have been improved. The learning framework of the session was very much in the spirit of Povey and Angier's educative assessment [1]. GeoGebra is a very accessible and effective software, providing an environment for students to observe, articulate, think and do mathematics.

## References

1. Povey, H. and Angier, C. (2007), Creating opportunities for authoritative knowing: Some undergraduates' experiences of mathematics assessment. *Philosophy of Mathematics Education Journal* No. 21. Accessed via <http://people.exeter.ac.uk/PErnest/pome21/index.htm> [Accessed 5 March 2009].
2. Sangwin, C.J. (2009), Geometrical functions: tools in GeoGebra. *MSOR Connections* Nov 2008, Vol 8 No 4:18-20. Accessed via [http://www.mathstore.ac.uk/headocs/84Sangwin\\_C.pdf](http://www.mathstore.ac.uk/headocs/84Sangwin_C.pdf) [Accessed 5 March 2009].