The new AS and A Levels in Mathematics

Roger Porkess
Mathematics in Education and Industry (MEI)

rp@mei.uk.com

New AS and A Level syllabuses (now called specifications) were introduced in all subjects for first teaching in September 2000, under the banner Curriculum 2000.

There was very little change in the content of AS and A Level Mathematics, just a slight increase, but the surrounding regulations dealing with re-sits, access to formulae and use of calculators were all made more severe. More serious were changes to the environment in which mathematics was embedded; it was now to be one of 4 rather than 3 subjects in the first year, and students were expected to take 3 modules (now called units) in that year where two had been customary.

The combined effect of all these changes can only be described as a disaster for mathematics. Students no longer had sufficient time. The first Curriculum 2000 students took AS in 2001. The results for this cohort and for the subsequent two years are summarised in Table 1.

	2001		2002		2003	
Subject	**Rank**	**Fail (%)**	**Rank**	**Fail (%)**	**Rank**	**Fail (%)**
Welsh	1	2.8	1	2.2	1	0.9
Classics	2	4.0	2	4.2	3=	4.7
Express Arts	3	4.3	3	4.4	3=	4.7
Music	4	5.2	9=	8.5	8=	8.0
English	5	5.3	5	6.2	5	5.3
History	6=	6.4	8	7.9	7	7.2
Media St	6=	6.4	4	5.9	6	5.1
French	11=	9.0	16=	10.4	14	8.8
Spanish	11=	9.0	14=	9.9	11	8.4
Geography	13	9.2	14=	9.9	16	9.2
Business St	20=	13.0	21	13.1	21	13.2
Chemistry	20=	13.0	22	13.3	22	14.4
Sociology	22	13.5	23	14.8	23	14.8
Physics	23	13.9	24	15.4	25	16.0
Biology	26	15.6	26	17.1	27	18.3
Psychology	27	17.2	27	17.3	26	18.0
General St	28	18.1	28	19.9	29	19.3
Computing	29	19.5	30	21.7	31	21.9
Law	30	20.5	29	20.6	28	18.7
Mathematics	31	28.6	31	22.1	30	19.9

Table 1 AS Rankings by subject, 2001-3

As a result of their experience at AS Level, many students gave up mathematics in the summer of 2001 and so there was a fall of over 12 000 in the numbers going on to take A Level in 2002, as shown in Table 2.

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Year	Total candidates	Mathematics candidates	Mathematics percentage
1989	661591	84744	12.8
1990	684117	79747	11.7
1991	699041	74972	10.7
1992	731024	72384	9.9
1993	734081	66340	9.0
1994	732974	64919	8.9
1995	725992	62188	8.6
1996	739163	67442	9.1
1997	777710	68880	8.9
1998	794262	70554	8.9
1999	783692	69945	8.9
2000	771809	67036	8.7
2001	748866	66247	8.8
2002	701380	53940	7.7
2003	750537	55917	7.5

Table 2 Mathematics A Level Numbers and % of entries

Table 1 shows that the failure rate in AS Mathematics was considerably less in 2002, but still higher than in any other subject; although not quite bottom of the table in 2003 mathematics was still right at the tail. Table 2 shows a modest increase in absolute A Level numbers in 2003, but to nowhere near their level before Curriculum 2000, and, more worryingly, a decrease in the proportion of mathematics to other A Levels to its lowest ever level.

The changes to AS and A Level Mathematics, that are described in the rest of this article, are a response to this situation and represent a serious attempt to raise the numbers taking mathematics in sixth form to previous levels.

The timetable for these changes is given in Table 3.

Event	First time
Teaching	September 2004
AS Level certification	Summer 2005
University application	Autumn 2005
A Level certification	Summer 2006
Students at university	Autumn 2006

Table 3 Timetable for changes

Changes to AS and A Level Mathematics

This section deals with AS and A Level Mathematics. It does not cover Further Mathematics which comes in a later section.

Pure units

At present A Level Mathematics consists of 6 units; usually 3 of these units are pure and 3 applied. In the new scheme it will still consist of 6 units, but 4 of them will be pure and 2 applied.

The four pure units will be known as C1, C2, C3 and C4, C standing for "Core". The total content of C1 to 4 will be essentially the same as that in the units Pure Mathematics 1, 2 & 3 in existing specifications; it is the A Level common core. Thus the content of three current units is to be spread among four in the new scheme.

The content of the various applied units will be much as it is at present, but an A Level student will take 2 rather than 3 of them. Thus the 6 units making up a new A Level will cover the same content as 5 units in present A Levels. There will be a loss of one applied unit of content.

The new AS Mathematics will be half of the A Level, i.e. 2 pure units and 1 applied. The pure units will be C1 and C2 and they will cover the AS Level common core.

There will be differences between the various specifications in the way the AS core is divided between C1 and C2, but not in the two taken together. Similarly the division of material between C3 and C4 will vary but, put together, they will cover the same core material whatever the specification. Thus there will be rather less variability in the pure mathematics that students are meant to know.

Applied units

For most specifications the 2 applied units will consist of one of the pairs in Table 4.

Mechanics 1	Mechanics 2
Statistics 1	Statistics 2
Decision Mathematics 1	Decision Mathematics 2
Mechanics 1	Statistics 1
Mechanics 1	Decision Mathematics 1
Statistics 1	Decision Mathematics 1.

Table 4 Applied Mathematics combinations in A Level Mathematics

This is a change. At present there are many more than these 6 ways of putting together A Level Mathematics. It represents a substantial simplification both of what is available and of how it will be processed by the examination boards.

There can only be one applied unit in AS Mathematics and it must be one out of Mechanics1, Statistics 1 and Decision Mathematics 1. The other two units are C1 and C2.

AS and A2 units

The list of pairs of 6 possible applied units given above for A Level Mathematics is only possible because of a substantial concession to Mathematics in the general regulations.

Look at the combination C1, 2, 3 & 4, Mechanics 1 & Statistics 1. C1 and C2 are both AS units, as are Mechanics 1 and Statistics 1. So there are 4 AS units and just 2 A2 units, C3 & C4.

All other A Levels must have 3 AS units and 3 A2 units but uniquely mathematics is now being allowed either 4 AS and 2 A2 . That makes it possible for a candidate to study two applied strands.

It is also possible for a candidate to offer just one applied strand, for example C1 to 4 and Statistics 1 & 2. In that case the A level contains 3 AS units (C1, C2 and S1) and 3 A2 units (C3, C4 and S2).

This concession for mathematics came late in the day. Most examination boards were planning, albeit unenthusiastically, to offer AS and A2 examinations on the three units, Mechanics 1, Statistics 1 and Decision Mathematics 1 so that students would have done one strand at AS in Year 1 and another at A2 in Year 2. Apart from the expense of setting extra examination papers, there was the near certainty that some candidates would have ended up taking the wrong one. Now there will be just the one examination on each of these units.

Changes to Further Mathematics

Another major change will occur in the arrangements for Further Mathematics. At the moment there is a limit of one AS unit for any Further Mathematics qualification, be it AS or the full A Level. In the new scheme, AS Further Mathematics may contain up to 3 AS units.

This may sound a bit bureaucratic but it is the symptom of some real thought on behalf of QCA about how Further Mathematics may best be delivered. At the moment there are two related problems.

- Students starting their first year of A Level after GCSE do not know enough mathematics to make a start on Further Mathematics.
- Consequently it is common to separate off the Further Mathematicians for all their mathematics lessons and then accelerate them through the early work.

Thus a small group, quite possibly fewer than 5, are receiving not just the Further Mathematics lessons but those for the single Mathematics as well. Many schools cannot afford to do this.

However the new scheme is designed to allow work on the Further Mathematics to begin in parallel with that for the single A Level. Here is an example of how it will work.

Juliette	Level	Mathematics	Further Mathematics
Year 1	AS	C1, C2, S1	FP1, M1, D1
Year 2	A Level	C3, C4, S2	FP2, FP3, M2

Table 5 Mathematics & Further Mathematics, an example

Juliette takes Mathematics and Further Mathematics at both AS and A Level. In both years she attends the same class as everyone else for the Mathematics but, along with a few others, has special extra lessons for Further Mathematics. Her school has decided that those taking the single Mathematics A Level will take Statistics 1 & 2 as well as the compulsory units C1 to 4. In her Further Mathematics she does Further Pure Mathematics 1, Mechanics 1 and Decision Mathematics 1 for AS in the first year, and then follows these with Further Pure Mathematics 2 & 3 and Mechanics 2 in her second year.

A key element in the new arrangements is the unit Further Pure Mathematics 1, FP1. This is a compulsory AS Further Mathematics unit and replaces the present Pure Mathematics 4, an A2 unit.

The design restrictions on FP1 are tight.

- It cannot depend on anything covered in the A2 core (i.e. on C3 and C4).
- It cannot duplicate anything in C3 and C4.
- It must be intrinsically worthwhile.

In effect it is an alternative route into post-GCSE Mathematics, and there is just enough material to make a viable unit. Table 6 gives some possible topics.

Simple complex numbers
Summation of series (eg $\sum r^2$)
Proof, including some induction
Curve sketching for rational functions (but no turning points)
More algebra (inequalities, roots of equations)
Matrices

Table 6 Possible topics for FP1

We have had Further Mathematics since the start of A Levels, 50 years ago. During that time people have often asked whether "Further" means broader or deeper. We are now closer to an answer: at AS, on the whole, it means broader, at A2 deeper.

The example of Juliette will not be the only way for delivering Further Mathematics. It is, for example, common for those currently offering AS Further Mathematics to take 4 units in their first year and 5 in their second, and no doubt such patterns will continue. However, the really exciting thing is that on top of what is already in place there are genuine new opportunities.

AS Further Mathematics

While the new AS Further Mathematics is to be welcomed as a means of encouraging more students to do more mathematics, university admissions tutors should be aware of two points.

- It will not, in general, be possible to certificate AS Further Mathematics at the end of one year, i.e. prior to completing the full single A Level Mathematics at the end of the second year. The two would interfere with each other. At the time most students fill in their UCAS forms, they will know their uniform marks and so their likely grades but it will not be absolutely certain which units will be allocated to Mathematics and which to Further Mathematics. They will normally only receive their AS Further Mathematics certificates at the end of their second year.
- Many students will complete the 6 units for AS Mathematics and AS Further Mathematics at the end of their first year. Some will be tempted to give up mathematics at that point, without completing the single A Level Mathematics. They may argue that universities will regard 2 AS Levels as equivalent to an A Level. For those going on to read Mathematics or related subjects, that would be a disaster. They will not have covered the A2 part of the A Level core and so there are many topics they will know nothing about, for example the product and quotient rules and integration by substitution.

Other qualifications

At present many qualifications other than Mathematics and Further Mathematics may be obtained from suitable combinations of units, for example AS Applied Mathematics and AS and A Level Statistics. It was QCA's intention to cut out all such alternative qualifications, and allow only AS and A Levels in Mathematics and Further Mathematics.

However as a result of lobbying they relented in the case of Pure Mathematics and this will be available as a qualification at both AS and A Level. A large uptake is not expected.

There will be AS, and possibly A Level, qualifications in Statistics but these will be awarded from different suites of units from Mathematics; at the most Statistics 1 will be common.

Verdict

When it was announced that there would be changes, the expectation was that they would be small, a slimming down to make AS Level easier for students to cope with in their first year. Instead what has emerged is probably the most far-reaching reform in the history of A Level Mathematics. Will it work ? Let me end with a personal view.

We have lost the 50-50 balance that has always existed between pure and applied mathematics. This seems a terrible shame. For many people the subject lives through its applications, and what they have learnt in those strands (for example the statistics) proves to be the most useful part of the subject when they go to university. Undoubtedly something had to be done, but I would have preferred to see a uniform trimming across all the strands; others did not agree.

That, however, was last year's argument and there is no point in going back on it. We must now all make the best of the situation that follows on from the decisions that were made.

As I have explained the content of a new A Level will be 5/6 of that of an existing A Level. There will be those who try to claim that this is not a diminution in standards. I am not one of them. There will be new A Levels and old A Levels and a new A Level will not mean as much as an old one.

However to argue in terms of standards is to miss the point. For a variety of reasons, many of them outside mathematics, the present A Level Mathematics is manifestly no longer fit for purpose. We now have half the number of people doing mathematics that we had 20 years ago. It is seen as more demanding than other subjects, both in terms of difficulty and the amount of its content. We need many more students taking mathematics post-16 and the present A Level is preventing this happening.

There is already no lack of complaints from universities about the poor mathematics of their entrants. How will these reforms help the situation?

The "official" answer is that by allowing students more time to do the pure mathematics, they will be better at it and so better generally. I am sure this will be true for many of the weaker students, but I am sceptical of the argument when it comes to the more able.

However, the good news is that the new arrangements for Further Mathematics should make it possible for able (and reasonably able) students to end up knowing more mathematics than they do at present. There should no longer be any excuse for suitable students not to have access to Further Mathematics, at least at AS Level, either from their own schools or through the successful MEI distance learning scheme, now expanding across the country.

Universities should recognise that those coming in with a new single A Level will not have met as much mathematics as their current counterparts, but that if they have also taken a new AS Further Mathematics they will definitely know more. There will be a change in the balance between Mathematics and Further Mathematics and university offers need to reflect that change.

The argument that it is wrong to mention Further Mathematics in an offer because it is not accessible to all students is no longer valid.

Replacing an offer of, say, "B in Mathematics" with "Either A in Mathematics or B in Mathematics + B in AS Further Mathematics" would, in my view, be an entirely appropriate response. It would carry a very clear message to schools' managements that in this new situation Further Mathematics is even more important. That would lead to better future undergraduates.

If, on the other hand, universities carry on as though nothing has happened, they run the risk of undermining the balance in a package of measures that have the potential to be very good news.

So, good or bad ?

The answer may well lie in the hands of admissions tutors.