Technical Paper

The Introspective Orifice Meter Uncertainty Improvements

Allan Wilson, Accord ESL Phil Stockton, Accord ESL Richard Steven, DP Diagnostics

1. INTRODUCTION

1.1 Overview

At the 2019 NSFMW, the authors presented: 'Data Reconciliation In Microcosm - Reducing DP Meter Uncertainty' [1]. Mathematical techniques, based on steady state data reconciliation, were developed to improve the performance of flow meters, including fine adjustments to the stated flowrate prediction while lowering uncertainty. These techniques were collectively described under the term: 'Maximum Likelihood Uncertainty' (MLU).

MLU requires multiple instrument readings. In the case of differential pressure (DP) meters this is provided by axial pressure profile analysis facilitated by a third pressure tapping generating three differential pressure readings: primary DP (Δ Pt), recovered DP (Δ Pr), and permanent pressure loss (Δ Pl). Each of these differential pressures can be used independently to calculate the flow rate and each of these flow calculations has its own flow coefficient, denoted Cd, Kr and Kppl, respectively.

MLU, applied to DP meters, reconciles the three measured DPs so that the three resultant calculated flow rates equal one another (satisfying mass balances) and the recovered and PPL DPs sum to the primary DP (satisfying the DP balance). It does this in a statistically optimal fashion in accordance with the uncertainties in the measurement sensors and associated input parameters.

The 2019 paper applied data reconciliation techniques to a single set of flow meter measurements obtained simultaneously at a specific time. In effect this is 'steady state MLU'. This technique is now extended to take advantage of time, that is, the method is extended from a static to dynamic data analysis.

In essence, steady state MLU extracts the maximum information from the existing measurements in order to obtain optimal estimates of the system variables at one instant in time. Time provides an extra dimension in which repeated measurements by the same instruments generate additional information that can be exploited by the MLU techniques to improve the estimates of flow rate and further reduce its uncertainty.

For example, a DP meter that monitors the meter's axial pressure profile, has three flow equations using three flow coefficients. These flow coefficients are ostensibly constant in time this extra information can be incorporated into the MLU technique using the Kalman Filter. Kalman Filters are typically used to model dynamic systems where some relationship defines the evolution of the system state with time and updates the state with measurements. By analyzing multiple data grabs at different times, the Kalman Filter reduces the flow rate uncertainty and improves the estimation of the flow coefficients, thereby self-tuning the DP meter in-situ.

Technical Paper

This paper describes the extension of the MLU approach to include the time dimension by application of a Kalman filter to an orifice meter with three DP measurements. (It should be noted that the approach is applicable to any DP meter and not restricted to the orifice meter type). Throughout the rest of the document the approach is termed 'Kalman MLU'.

2 INTRODUCTION TO THE KALMAN FILTER

2.1 Overview of the Kalman Filter

The Kalman filter is extensively used in various sections of science and industry, e.g. guidance, control, and positioning of vehicles, signal processing, and econometrics [7], [8]. The Kalman filter is applied to dynamic systems and uses process models, along with measurements with statistical noise, to provide best estimates of variables in the system. It is an algorithm that uses a process model to predict how the system's variables and parameters propagate from one time step to the next and reconciles these with a series of measurements observed over time, containing statistical noise (i.e. uncertainty). It does this in a statistically optimal fashion and produces estimates of the variables and parameters that tend to be more precise than those based on measurements alone.

The algorithm works in a two-step process as indicated schematically in Figure 1.

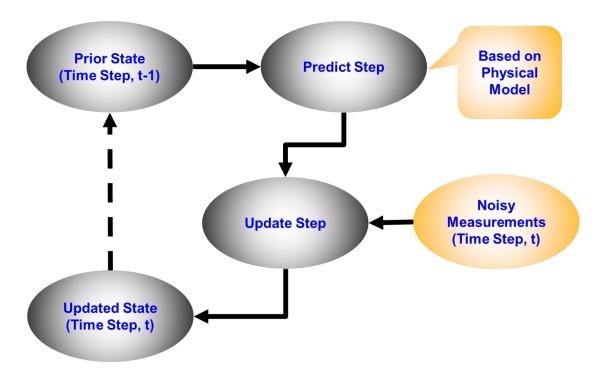


Figure 1 Kalman Filter Algorithm

In the prediction step, the Kalman filter produces estimates of the current state variables, along with their uncertainties. Once the outcome of the next measurement (necessarily corrupted with some amount of uncertainty, including random noise) is observed, these estimates are updated (in the update step) using

Technical Paper

a weighted average, with more weight being given to estimates with lower certainty. The algorithm is recursive in that it uses only the present input measurements and the previously calculated state and its uncertainty matrix; no additional past information is required.

In effect the Kalman filter is data reconciliation extended into the time domain. It exploits temporal dependencies using a model that describes how the system parameters and variables propagate from one time step to the next. It uses the same weighted least square uncertainties, as data reconciliation, to update the values of the parameters and variables in the system.

A number of extensions and generalised methods have been developed over the years for the Kalman filter, but the method proposed here is relatively simple, in that the full Kalman filter includes terms for control variables which are not required for this application to DP meters. A more complete description of the Kalman filter and its applications is provided in [2]. Additionally, the Kalman filter has been successfully applied to estimate well potentials on an offshore platform [6].

2.2 Application to Differential Pressure (DP) Flow Measurement Devices

The mathematics of steady state data reconciliation has previously been used by the authors to develop an approach to reduce the uncertainty associated with various measurement devices [1]. This was termed the MLU method.

An enhancement to this approach is to incorporate the time dimension. Hence, the use of the Kalman filter appeared a natural extension of the ideas used to develop steady state MLU.

Time provides an extra dimension in which repeated measurements by the same instruments generate additional information that can be exploited by the MLU techniques to improve the estimates of flow rate and further reduce its uncertainty.

It also allows the temporal dependencies of system parameters to be exploited. For example, a DP meter that monitors the meter's axial pressure profile, has three flow equations using three flow coefficients. These flow coefficients are ostensibly constant in time. This extra information can be incorporated into the MLU technique using the Kalman Filter. The various steps associated with this implementation of the Kalman filter for DP meters are described briefly below.

The flow coefficients display a weak variation with Reynolds number. If the flow rate experienced by a meter changed significantly, and hence the Reynold's number changed, then the above assertion that the flow coefficients remain constant would not be strictly true. However, for simplicity in this discussion, the weak variation with Reynolds number has been ignored, though this dependency will be incorporated in the full algorithm.

State Variables

The six state variables of the DP measurement system have been defined as:

- Primary or 'traditional' DP (ΔPt)
- recovered DP (ΔP_r)
- permanent pressure loss DP (ΔP_{PPL})
- modified discharge coefficient (C_d')

Technical Paper

- modified expansion coefficient (K_r')
- modified permanent pressure-loss coefficient (K_{ppl}')

The so-called modified coefficients (indicated by the " $^{\prime}$ " superscript) are derived from the more traditional coefficients C_d , K_r and K_{PPL} .

From ISO-5167 [4] and Steven [5] the 'primary' (or 'traditional'), 'recovered' and 'permanent pressure loss' mass flow rates $\dot{m_t}$, $\dot{m_r}$ and \dot{m}_{PPL} are given by equations (1), (2) and (3):

$$\dot{m_t} = EA_t Y C_d (2\rho \Delta P_t)^{1/2} \tag{1}$$

$$\dot{m_r} = EA_t K_r (2\rho \Delta P_r)^{1/2} \tag{2}$$

$$\dot{m}_{PPL} = AK_{PPL}(2\rho\Delta P_{PPL})^{1/2}$$
 (3)

Where

- D is the meter inlet diameter and the meter inlet area, $A = \frac{\pi D^2}{4}$
- d is the meter throat diameter and the meter throat area, $A_t = \frac{\pi d^2}{4}$
- E is the 'velocity of approach', $E = 1/\sqrt{(1-\beta^4)}^0.5$
- β is the 'beta', $\beta = \sqrt{(A_t/A)}$
- ullet \mathcal{C}_d is the discharge coefficient
- K_r is the expansion coefficient
- K_{PPL} is the permanent pressure-loss coefficient
- Y is the fluid's expansibility
- ρ is the fluid's density

At stable operating conditions, D, d, E, A, A_t , C_d , K_r , K_{PPL} and Y should all remain constant. Hence, modified coefficients can be defined to be:

$$C_d' = EA_t Y C_d \tag{4}$$

$$K_r' = EA_t K_r \tag{5}$$

$$K'_{PPL} = AK_{PPL} \tag{6}$$

Prediction Step

The first equation in the predict phase of the Kalman filter employs the transition matrix, which predicts how the state variables from the previous time step (t-1) propagate to the current time step (t).

For the case of the modified flow coefficients, the model assumes that they are constant throughout time, i.e.:

Technical Paper

$$C'_{d,t} = C'_{d,t-1} = C'_d (7)$$

$$K'_{r,t} = K'_{r,t-1} = K'_r \tag{8}$$

$$K'_{PPL,t} = K'_{PPL,t-1} = K'_{PPL} \tag{9}$$

The filter is provided with initial estimates of the values of these parameters and their uncertainty. However, it should be noted that their true value is not known, only that the true value remains constant in time. The Kalman filter uses this information and the measurements to update its estimate of these parameters and improve the uncertainties of those estimates.

For the case of the three differential pressures, these can vary unpredictably as dictated by the fluctuations in the process from one time step to the next.

The uncertainty, or more strictly the covariance, in the previous time step's state variables i.e. an output from the Update Step of the Kalman filter run at the previous time step (t-1), is used and propagated forward in time adding process noise. The process noise represents the uncertainty in the physical model.

For the case of the flow coefficients in this model, this process noise is zero since they remain constant. For the case of the differential pressures, as the flow and pressure can fluctuate around the mean unpredictably each has noticeable loggable process noise. Though the assignment of this process noise may appear somewhat arbitrary at this point, it is an adjustable parameter and its correct value can be ensured by monitoring the innovation and auto-correlation statistics output by the Kalman filter.

Update Step

This step updates the state variables using the data from available measurements. It also imposes the mass and pressure balance constraints associated with the system.

The available measurements are the three differential pressure measurements. The DP measurements are necessarily uncertain and hence the Kalman filter's estimates of the values of the flow coefficients are updated using a weighted average of all estimates and measurements, with more weight being given to those with lower uncertainty. The constraints are also imposed, and these adjust both the coefficients and measured differential pressures, to ensure they are complied with.

As previously described in [1], the constraint equations are:

$$C'_{d,t}(2\rho\Delta P_t)^{1/2} - K'_{r,t}(2\rho\Delta P_r)^{1/2} = 0$$
 (10)

$$C'_{d,t}(2\rho\Delta P_t)^{1/2} - K'_{l,t}(2\rho\Delta P_l)^{1/2} = 0$$
(11)

Technical Paper

$$K'_{l,t}(2\rho\Delta P_l)^{1/2} - K'_{r,t}(2\rho\Delta P_r)^{1/2} = 0$$
(12)

for the mass balances, and

$$\Delta P_t - \Delta P_r - \Delta P_{PPL} = 0 \tag{13}$$

for the pressure balance.

Because the uncertainty of the measurements and the process model uncertainty (process noise) may be difficult to determine precisely, it is common to discuss the filter's behaviour in terms of gain. The Kalman gain is a function of the relative uncertainty of the measurements and current estimates of the flow coefficients. These can be tuned to achieve particular performance. With a high gain, the filter places more confidence on the measurements, and thus follows them more closely. With a low gain, the filter follows the model predictions more closely, smoothing out noise but decreasing the responsiveness. The gain is adjusted using the process noise described in the Predict Step.

The uncertainties of the state variables, i.e. the updated coefficients and DPs, are also filter outputs. These are in the form of a covariance matrix which is a familiar feature in data reconciliation techniques. This covariance matrix, along with the uncertainty in the fluid density, is then used in the determination of the uncertainty in the calculated mass flow rate at each time step.

Recursion

The filter's calculated state variables and their associated uncertainties at the current time step, t, form the inputs to the calculations of the next time step, t + 1. This is the recursive nature of the Kalman filter, and all the information required to perform the calculations in the next time step is contained within the estimates and uncertainties from the current time step.

This feature makes the Kalman filter computationally very efficient, which is attractive from a practical software implementation viewpoint because the algorithm is concise and only requires input from the previous time step.

The Kalman MLU method proposed here is termed an Extended Discrete Kalman filter in the literature [3]. The Extended Kalman filter is required to handle the non-linear constraints, see Equations 10, 11 and 12, and its discrete nature is due to it being applied at discrete time intervals rather than continuously (though the time intervals are short in duration).

3 THOUGHT EXPERIMENT USING A THEORETICAL METER

3.1 Motivation

Since orifice plate meters are not normally calibrated it is difficult to assess the performance of the algorithm against a reference device. Hence, in this initial theoretical example, the construction of a hypothetical meter in which a constant 'true' mass flow rate is assigned, and hence known, provides such a reference device albeit theoretical.

Technical Paper

The meter parameters are also assigned: pipe diameter (D), throat diameter (d), fluid density, flow coefficients, etc. From this data, the associated 'true' values of the three differential pressures can be back calculated. The 'true' differential pressures can then be assigned realistic measurement uncertainties. That is, randomly generated, pseudo-measured differential pressure values can be produced consisting of the constant theoretically calculated DPs with superimposed random variations within their respective allotted uncertainty ranges. These can be calculated for each time step. These vary around the 'true' value in accordance with their uncertainties. In effect this mimics the three sets of DP measurement data available at each time step encountered with a real meter in the field.

In addition, to mimic real meters the theoretical set values for the meter geometry and fluid properties need realistic uncertainties assigned,—which again can be used to generate pseudo-measured values, but these stay constant in time.

This then is a representation of the data associated with a real meter measuring a constant mass flow rate. The advantage of this approach is that the true flow rate is known and can be compared against the predictions of flow produced using the traditional flow equations and the Kalman MLU based approach which will be derived from the differential pressures and meter properties corrupted by realistic noise.

3.2 Meter Description

This hypothetical example is based on a 4", sch 40, 0.5 beta orifice meter with an inlet diameter of 0.102 m and throat diameter of 0.0508 m. A drawing of the hypothetical meter run is shown in Figure 2. Measured input variables, assigned relative uncertainties and associated absolute uncertainties are listed in Table 1.

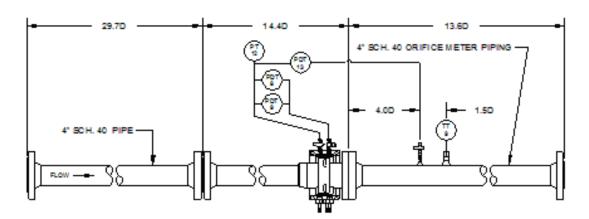


Figure 2 Hypothetical 4", sch. 40 Orifice Meter Run

Table 1 – Hypothetical 4", 0.5 Beta Orifice DP Meter Variable and Parameter Uncertainties

Variable /	Unit	'True' Value	Percent	Absolute
Parameter			Uncertainty	Uncertainty
Mass Flow	kg/s	3.2064		
DPt	Pa	90,796	1.0%	908
DPr	Pa	23,931	1.0%	239

Technical Paper

DP _{PPL}	Pa	66,866	1.0%	669
d	М	0.0508	0.05%	0.000025
D	m	0.102	0.25%	0.00026
Υ	Dimensionless	0.991	0.30%	0.0030
C_d	Dimensionless	0.602	0.5%	0.003
Kr	Dimensionless	1.163	2.9%	0.017
K _{PPL}	Dimensionless	0.177	1.2%	0.002
ρ	kg/m³	36.304	0.27%	0.098

The Reader-Harris Gallagher equation [4] was used to calculate the discharge coefficient C_d and the same reference used to obtain the uncertainty in C_d . The values and uncertainties associated with K_r and K_{PPL} were estimated based on C_d and the pressure loss ratio.

One hundred time steps were then simulated in which pseudo-measured values of the three DPs were randomly generated in accordance with their uncertainties around the true values in the table above. Using this measurement data through time, the Kalman MLU algorithm was employed to obtain optimal estimates of the state variables (DPs and modified flow coefficients) at each time step. This also allowed the mass flow and its associated uncertainty to be calculated at each time step.

In Figure 3, the mass flow rates calculated via these pseudo-measured DP values using both the Traditional equation (1) and the Kalman MLU approach are compared against the example's constant 'True' value.

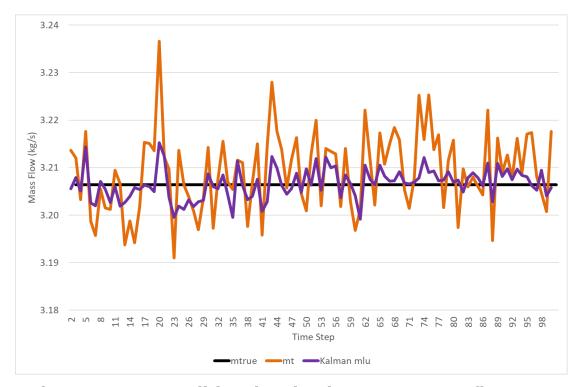


Figure 3 True, Traditional and Kalman MLU mass flow rate versus time step

Technical Paper

The flows are plotted against time steps. These time steps would typically be three second intervals (say) when grabs of data obtained from orifice meter are reconciled. One hundred time steps are shown for illustrative purposes.

The example's assigned 'true' mass flow remains constant. The values calculated using the Traditional flow equation with no Kalman filter MLU, represented by the orange line vary at each time step in accordance with the variation in DP_t . The magnitude of the deviations from the true value do not reduce with time.

The flow rate prediction with the Kalman MLU applied, continuous purple line, also deviates from the true value but the deviations are lower than observed with the standard Traditional flow equation. Also, by visual inspection the Kalman MLU deviations reduce with time as the uncertainty in its flow estimates improve.

The uncertainty in the mass flow rate calculated according to AGA 3 [9], referred to as the 'Traditional' uncertainty (mt ε % dashed orange line) is compared against the applied Kalman MLU uncertainty (Kalman MLU ε % purple line) in Figure 4:

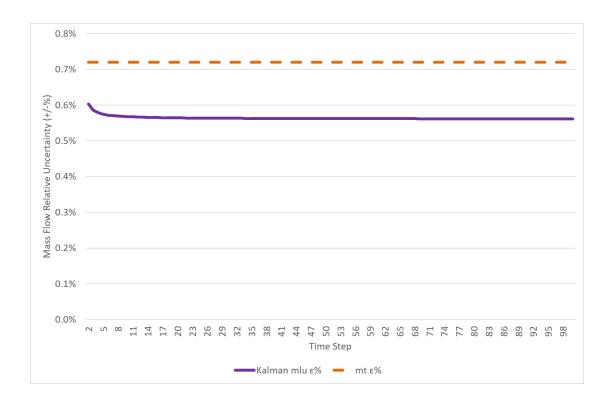


Figure 4 Traditional and Kalman MLU mass flow rate uncertainties versus time step

Figure 4 shows an immediate reduction in mass flow rate uncertainty. This is the uncertainty reduction experienced using the steady state MLU as presented in [1]. The Kalman MLU uncertainty then further reduces with time as it accumulates more data and improves its estimate of the mass flow rate. In contrast the traditional flow uncertainty is static as the 'True' flow rate is not varying.

The next three plots (Figure 5, Figure 6 and Figure 7) show the evolution of the Kalman MLU estimates of the modified flow coefficients with time. These plots are

Technical Paper

one sample from a whole host of randomly generated simulations in which the starting value of the coefficients are randomly deviated in accordance with their uncertainties.

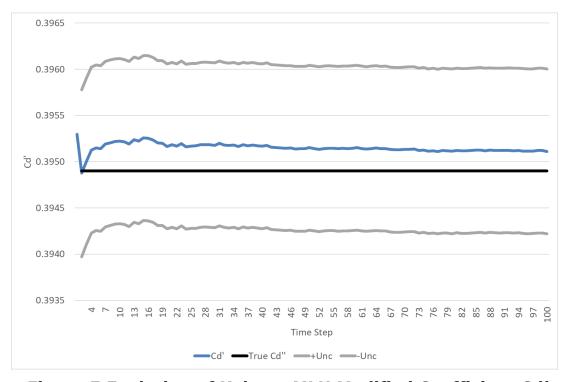


Figure 5 Evolution of Kalman MLU Modified Coefficient Cd' versus time step

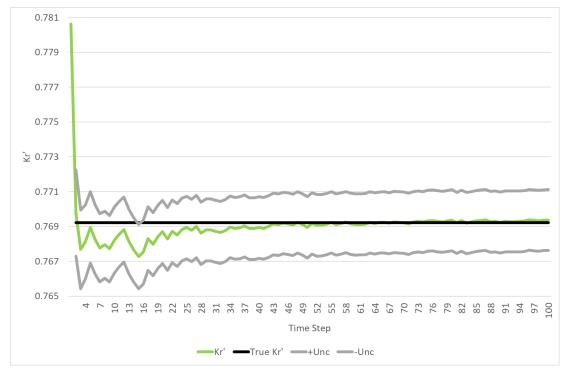


Figure 6 Evolution of Kalman MLU Modified Coefficient Kr' versus time step

Technical Paper

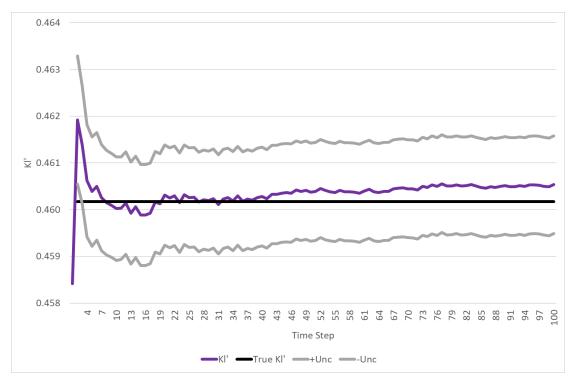


Figure 7 Evolution of Kalman MLU Modified Coefficient Kl' versus time step

These plots are presented to show the potential of the Kalman Filter MLU approach to self-tune the flow coefficients. It should be highlighted that the above plots are for one simulation of a meter and the results vary depending on the initial starting values.

Similarly, the uncertainty in the three reconciled differential pressures also improves with time and this is illustrated in Figure 8, Figure 9 and Figure 10:

Technical Paper

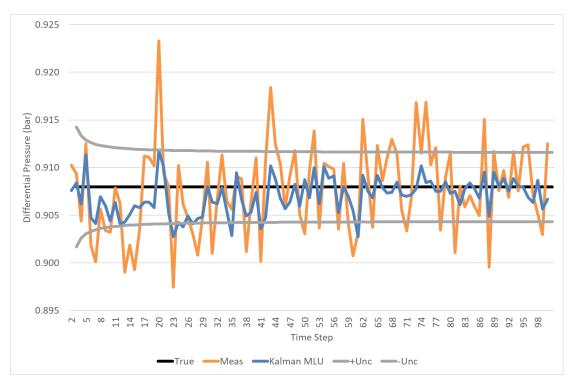


Figure 8 Evolution of Kalman MLU Reconciled Differential Pressure DP_t versus time step

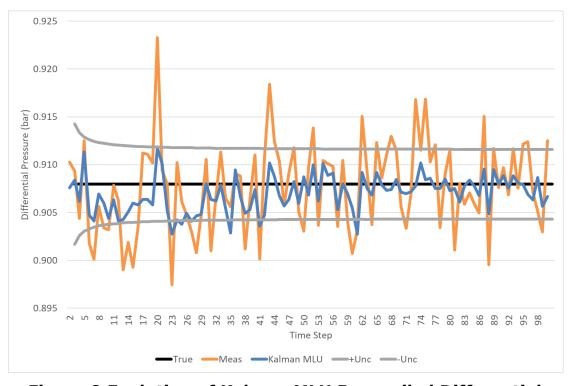


Figure 9 Evolution of Kalman MLU Reconciled Differential Pressure DP_r versus time step

Technical Paper

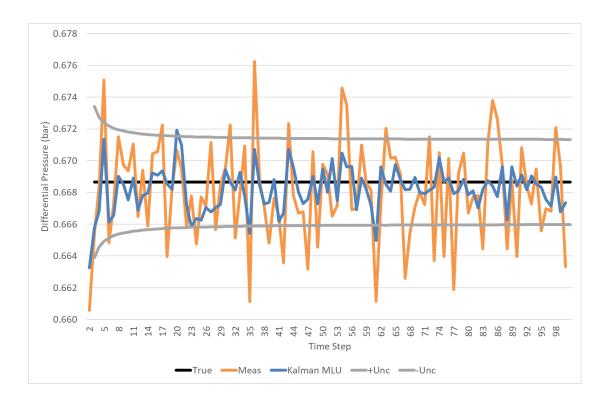


Figure 10 Evolution of Kalman MLU Reconciled Differential Pressure DP_{PPI} versus time step

It is evident, by visual inspection, in all three plots that the reconciled differential pressures are closer to the true values than the measurements. The 95% uncertainty bands of the reconciled values are also plotted (grey lines) about the true value and this illustrates how the uncertainties in the reconciled DPs reduce with time.

The above analysis demonstrates the theoretical feasibility of the Kalman MLU approach and its ability to reduce measurement uncertainty in comparison to standard meter system uncertainty calculation as shown by AGA 3 [9]. The reduction in uncertainty with time also illustrates that the Kalman MLU approach is an improvement on the steady state MLU.

4 APPLICATION TO REAL DATA

4.1 Introduction

The chaos and noise of real-world data presents a more formidable test of the MLU Kalman filter than any theoretical data in assessing the feasibility and robustness of the method.

The problem with real data is that we can never know the true values of all the variables that we are estimating, and the performance of the filter has to be assessed against more limited data and the use of engineering judgement.

The data obtained in this example was from an orifice meter that was installed in a test facility and through which gas was flowed. The meter parameters and fluid

Technical Paper

properties were known, and the three differential pressures measured and recorded at 3 second intervals. The test facility had a reference meter from which the instantaneous mass flow rates were recorded.

The approach in Section 3 for the hypothetical meter was applied to this real data - the differences being that:

- the 'true' constant mass flow rate is replaced by a mass flow rate obtained from a reference meter, with a significantly lower uncertainty (±0.5%) than the orifice meter under test (±0.75%), and a flow that is fluctuating in time;
- the randomly generated DP measurement values, based on known true values and their uncertainties, are replaced by real DP measurements whose true values are not precisely known and whose uncertainties are only known to a nominal level.

4.2 Meter Description

This real example is based on a 6", 0.6077 beta orifice meter with an inlet diameter of 0.1463 m and throat diameter of 0.0889 m (see Figure 11). Measured input variables, relative uncertainties and absolute uncertainties are listed in Table 2.



Figure 11 6", 0.6β Orifice meter at Test Facility with DP_t, DP_r, and DP_{PPL} with Field Mount Flow Computer

Technical Paper

Table 2 - 6", 0.6 Beta Orifice DP Meter Variable and Parameter Uncertainties

Officer tallities								
Variable /	Unit	Measured	Percent	Absolute				
Parameter		Value	Uncertainty	Uncertainty				
Mass Flow*	kg/s	11.299						
DP _t *	Pa	100,546	1.0%	1,005				
DP _r *	Pa	37,359	1.0%	374				
DP _{PPL} *	Pa	63,109	1.0%	631				
d	m	0.0889	0.05%	0.000045				
D	m	0.146	0.25%	0.00036				
Υ	Dimensionless	0.994	0.30%	0.0030				
C_d	Dimensionless	0.599	0.5%	0.003				
Kr	Dimensionless	0.982	1.8%	0.018				
K _{PPL}	Dimensionless	0.300	1.2%	0.003				
ρ	kg/m³	39.807	0.27%	0.108				

^{*} These are typical average values.

In Figure 12, the mass flow rates calculated using the Traditional equation without the Kalman MLU approach and those calculated using the Kalman MLU approach are compared against the Reference meter values.

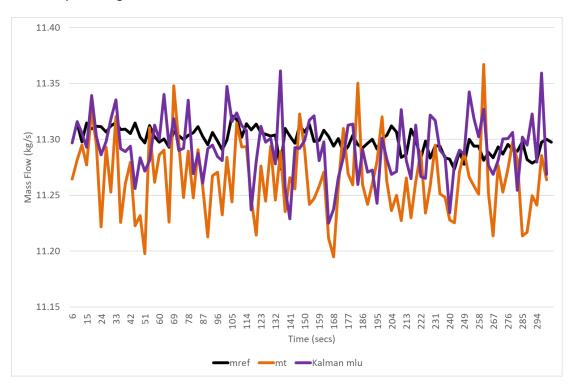


Figure 12 Reference, Traditional and Kalman MLU mass flow rate versus time

The Reference meter flow indicated by the black line, varies, but is more stable than either the Traditional or Kalman MLU values. The Kalman MLU values are on average closer to the reference meter and exhibit less variability than the Traditional flow. This is illustrated more clearly in Figure 13 in which the cumulative mass (mass flow integrated over time) difference with the Reference meter flow is plotted:

Technical Paper

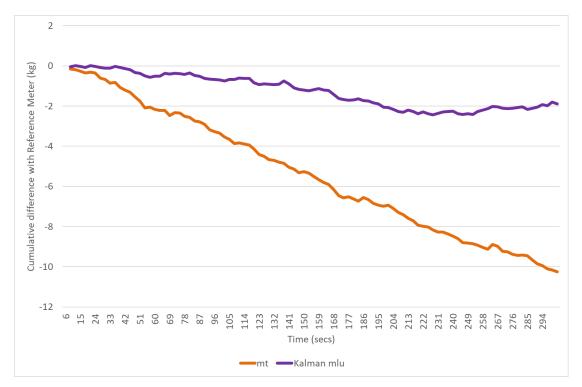


Figure 13 Traditional and Kalman MLU cumulative mass difference with Reference Meter versus time

The 'Traditional' mass flow rate uncertainty (mt ϵ % dashed orange line) is compared against the Kalman MLU uncertainty (Kalman MLU ϵ % continuous purple line), in Figure 14:

Technical Paper

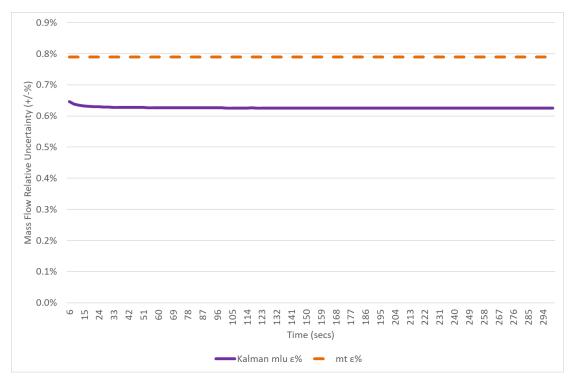


Figure 14 Traditional and Kalman MLU mass flow rate uncertainties versus time step

As observed in the theoretical example, this real data illustrates that the Kalman MLU uncertainty reduces with time as it accumulates more data and improves its estimate of the mass flow rate. In contrast the traditional flow uncertainty is relatively static as the Reference flow rate remains roughly constant.

The next three plots (Figure 15, Figure 16 and Figure 17) show the evolution of the modified flow coefficients with time. The true values are not known but these plots illustrate how the Kalman filter adjusts them to stable and consistent values.

Technical Paper

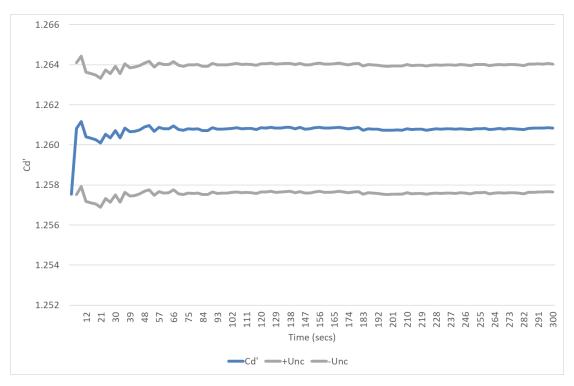


Figure 15 Evolution of Kalman MLU Modified Coefficient Cd' versus time step

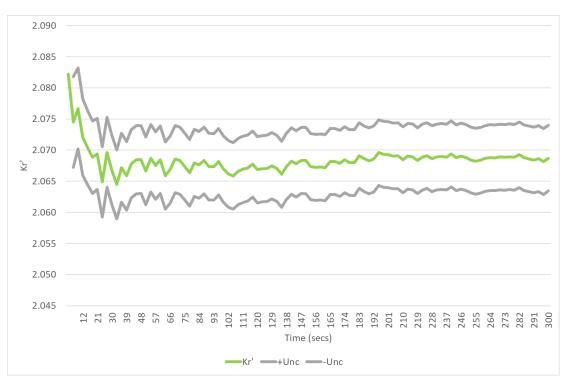


Figure 16 Evolution of Kalman MLU Modified Coefficient Kr' versus time step

Technical Paper

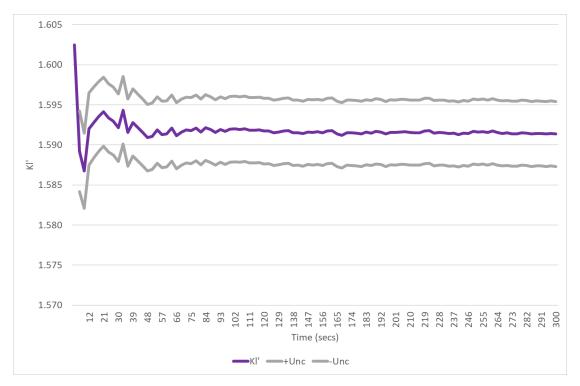


Figure 17 Evolution of Kalman MLU Modified Coefficient Kl' versus time step

5 CONCLUSIONS

The DP meter operating principle, the associated flow rate calculation algorithms, and their output uncertainties are well established and have not been substantially developed or changed for many years.

However, the advent of modern digital instrumentation offers far more detail on the nature of primary signals being read than was historically available. This includes both a signal's central tendency and its variation. Presently, any variation of a DP meter's DP signal is usually lost as batches of signals over a time period are compressed into single mean values. However, that signal variation can contain extra information regarding the state of the system.

There is value, i.e. information, in the signal variation that is typically being ignored. The Kalman MLU Intellectual Property is a method of extracting that extra information, and a way of further reducing the DP flow meter's uncertainty. In effect, the Kalman MLU is a way of getting the DP flow meter system to 'consider' the meaning of the often ignored primary signal variations, a way of more deeply considering the state of the system, a way of creating a more 'introspective flow meter'.

Use of the Kalman MLU, is use of the often ignored primary signal variation, to improve both the DP flow meter's primary flow prediction and that prediction's uncertainty.

Technical Paper

6 REFERENCES

- [1] Allan Wilson and Phil Stockton (Accord ESL), Richard Steven, (DP Diagnostics), Data Reconciliation in Microcosm Reducing DP Meter Uncertainty, Proceedings of the 37th International North Sea Flow Measurement Workshop, 22-25 October 2019
- [2] Shankar Narasimhan and Cornelius Jordache, Data Reconciliation and Gross Error Detection, An Intelligent Use of Process Data, published in 2000 by Gulf Publishing Company, Houston Texas, ISBN 0-88415-255-3.
- [3] Applied Optimal Estimation, Arthur Gelb, 1974, The Analytical Sciences Corporation, ISBN 0262570483.
- [4] International Standard Organisation Measurement of Fluid Flow by Means of Pressure Differential Devices, Inserted in circular cross-sections running full, no. 5167 2003.
- [5] Steven R., Diagnostic Methodologies for Generic Differential Pressure Flow Meters, North Sea Flow Measurement Workshop 2008, St. Andrews, Scotland, UK.
- [6] Euain Drysdale, Phil Stockton, Could Allocation be Rocket Science? Using the Kalman Filter to Optimise Well Allocation Accuracy, Proceedings of the 33rd International North Sea Flow Measurement Workshop, 20-23 October 2015
- [7] Paul Zarchan; Howard Musoff (2000). Fundamentals of Kalman Filtering: A Practical Approach. American Institute of Aeronautics and Astronautics, Incorporated.
- [8] Ghysels, Eric; Marcellino, Massimiliano (2018). Applied Economic Forecasting using Time Series Methods. New York, NY: Oxford University Press.
- [9] AGA Report 3, Orifice Metering of Natural Gas and Other Related Hydrocarbon Fluids, 4th Edition, 2000